TRAVERSING THE INFINITE AND PROVING THE EXISTENCE OF GOD

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Abstract. The aim of this paper is to present a proof to the conclusion that is impossible to traverse an infinite series (in particular, an infinite series of past moments). This may also show (given additional assumptions) that the series of past moments cannot be infinite. In the first section I formulate five theses concerning traversing, successive addition and successive subtraction and I present the idea of the argument: if it were possible to traverse an infinite past, it should be in principle possible to go back, which is, however, impossible. The main body of the paper is concerned with working out a simple mathematical representation of some structural features of processes like traversing and successive addition. I also make a crucial distinction between completion of a process at a particular time and its timeless „completion" in infinite time. In section V, I present the formal proof and defend it against a possible objection of question-begging. Finally, I suggest that my argument can contribute to constructing arguments for God's existence, and to solving the problem of the asymmetry of our attitudes towards death and prenatal non-existence.

I. Introduction

In this paper I will present an argument to the effect that it is logically impossible that infinite time has been traversed.

The claim that it is impossible to traverse an infinite time traditionally served as a premiss in a cosmological argument for the existence of God. The argument based on this premiss was developed by mediaeval Islamic philosophers and in the West by St. Bonaventure. The argument can be presented in the following way:

(1) Everything that begins to exist has a cause of its existence.
(2) Suppose that the universe did not begin to exist.

(3) Then it would now have existed for an infinite time.
(3) Then an infinite time would have been traversed.
(4) Lemma: It is impossible to traverse an infinite time.
(5) Therefore the universe began to exist.
(6) Therefore the universe has a cause of its existence. (by (1),(5))
(7) This cause transcends the universe and is called 'God'.

In our times, William Craig did much to revive this sort of argument and presented several arguments to show that the past cannot be infinite (Craig 1979, pp. 61-153; Craig, Smith 1995, pp. 3-76, 92-107. It will be convenient to start our discussion by looking at one of his arguments:

(1) The temporal series of events is a collection formed by a successive addition.
(2) A collection formed by successive addition cannot be an actual infinite.
(3) Therefore the temporal series of events cannot be an actual infinite. (Craig 1979, p. 103).

While I find this argument rather persuasive, it has an important weakness. For Craig, 'successive addition' means a real process of the growing of the collection of past events with passing time. Yet in that case the argument will be immediately rejected by anyone who professes a B-theory of time. Nevertheless, some distinguished detensers emphatically assert that tenseless time is not *static* (Oaklander 1994, p. 346; Williams 1994, pp. 361f, 367). In *some* sense, there is succession and „movement" in time. Now, movement, succession, whatever their real nature, and successive adding of objects, share some basic mathematical features and can be mathematically represented in exactly the same way.[1] Hence, if some mathematical operations will be possible with regard to some series[2], it will show at the same time that the series is in principle *traversable* and that it *can* be formed by successive addition. Similarly for the impossibility of traversing, and of forming the series by successive addition. In fact, traversing can be mathematically represented by the operation of successive subtraction of elements from a set until an empty set is reached (I will call this operation 'exhausting by successive subtraction') or, equally well, by successive add-

[1] I will call all such processes 'gradual processes'.
[2] Throughout the article 'series' means no more than 'ordered set'.

ing of elements until a whole series is reached ('forming by successive addition'). I thus make two claims:

(P1) Set/series X can be traversed ≡ X can be formed by successive addition
(P2) Set/series X can be traversed ≡ X can be exhausted by successive subtraction

In the third part of my paper I will provide mathematical definitions of the right side of (P1) and (P2). For now, it is important to note that these principles bear no ontological commitment to time series being *in fact* formed by successive addition. I claim that if it can be traversed, it is such that it is *possible* to form it by successive addition. 'Possibility' refers here to the possibility of performing some simple mathematical operations on a mathematical representation of the time series. There is no question of any actual process of assembling a time series, which a detenser would regard as absurd.

Shifting the focus to mathematical features of models enables us to sidestep problems concerning the nature of time and processes altogether. I also believe that if the discussions about infinity are to be clear and fruitful, it is essential to give our intuitions a sound mathematical representation. I will thus try to give a mathematical interpretation of 'successive addition' and other terms, such that it will satisfy (P1) and (P2) and further principles which express our intuitive understanding of structural features of successive addition, traversing etc. These principles will also be the basic assumptions I shall use in my argument.

(P3) An (actually) infinite set/series cannot be formed by successive addition starting from a finite set.

This postulate leaves open the possibility of forming something when we start from an infinite set, or when we do not *start* at all. (P3) is an expression of the known fact that ω, the first infinite set (which can be thought of as the set of all natural numbers) is not a successor of any smaller ordinal number.

The crucial idea for my argument is expressed by the two following principles, which seem to me self-evident:

(P4) Set/series X can be formed by successive addition ≡ X can be exhausted by successive subtraction
(P5) If a series can be traversed in one direction, it can be traversed in the opposite direction.

Subtraction is an inverse of addition. If we can build something up by adding elements step by step, then if we remove them *in the reverse order*, we will eventually dismantle it. And if we can exhaust something by removing elements one by one, then by adding them one by one we can restore it. Why then is (P5) needed at all, if we could represent traversing by addition or subtraction, given (P1) and (P2)? It is because (P4) does not say anything about the reversal of order or direction. Yet a thought about the order or direction is involved in our intuitive understanding of successive addition and subtraction, which is expressed by (P4), and I hinted at this a while ago. We could thus formulate a principle similar to (P5) about successive addition.[3] I have chosen (P5) because it simple and clear on its own.

I will show that it follows from (P1)-(P5) that if T* is infinite then it cannot be traversed. Let us consider the infinite series of past moments $T^* = \{\dots t_{-2}, t_{-1}, t_0\}$. A „moment" can be defined as a one-second interval. The argument in the informal mode runs as follows:

(1) Assume that the infinite series of past moments T* has been traversed in one direction.
(2) T* can be exhausted by successive subtraction. [(1), (P2)]
(3) If a series can be traversed in one direction, it can be traversed in the opposite
direction. [P(5)]
(4) T* can be traversed in the opposite direction. [(1), (3)]
(5) Traversing in the opposite direction of a series which can be exhausted by successive subtraction can be mathematically represented by an inverse operation. This operation is forming the series by successive addition, starting from an empty set.
(6) But T* is equivalent to the set of natural numbers ($T^* \cong \omega$), hence it cannot be formed by successive addition starting from an empty set. [(P3)]
(7) Therefore T* cannot be traversed in the opposite direction. [(5), (6), (P1)]
(8) Therefore T* cannot be traversed, contrary to the assumption. [(3), (7)]

Contradiction (1), (8).

□

[3] See the principle (P5*) below, section IV.

If one makes an additional assumption that it is a necessary property of the series of past moments that it can be traversed, one can infer that it cannot be infinite.

II. Completion in time

Before I go on to work out a mathematical representation of gradual processes, I need to clarify one important issue: what is meant by *completion* of the process *in* time.

We might start by looking at two celebrated passages from Russell: „when Kant says that an infinite series can „never" be completed by successive synthesis, all that he has even conceivably a right to say is that it cannot be completed *in a finite time.*" (Russell 1993, p. 161).

The second passage concerns Tristram Shandy's case. Tristram Shandy, the hero of G. Sterne's novel, writes an autobiography. What is peculiar about him is that it takes him a whole year to write about one day of his life. Russell writes: „Now I maintain that, if he had lived forever, and not wearied of his task, then, even if his life had continued as eventfully as it began, no part of his biography would have remained unwritten." (Russell 1937, p. 358).

It is worth noting how Craig paraphrases Russell's view: „Were he mortal, he would never finish, asserts Russell, but if he were immortal, then the entire book could be *completed*, since by the method of correspondence each day would correspond to each year, and both are infinite." (Craig, Smith 1995, p. 33).

With regard to this Smith rightly observes: „Rather, at no point in the past, and at no present, will Tristram Shandy's autobiography be *complete*." (Craig, Smith 1995, p. 88).

But further in his essay, Smith writes: „It may be the case that in a finite period of time this series cannot be written down, but it certainly is the case that it could be written down *in* an *infinite period* of time." (Craig, Smith 1995, p. 90).

Smith's use of the notion of completion in time is apparently incoherent. In the first quotation, he insists, rightly, that some processes cannot be completed *at any particular time*. Yet in the ensuing passages he is evidently ready to say that the result of the process is somehow reached, if there is a 1-1 correspondence between *all stages* of the process and the collection which is the result. Note that at least in some cases under discussion, like in the case of Tristram Shandy's collected works, the collection itself will not exist at any particular point in time – in its entirety it exist only from a *timeless* point of view on things. I submit, that there are in fact *two* notions of completion involved, only one of which deserves to be expressed by the phrase „completion in time". The distinction will not presuppose any view concerning the nature of time.

Let us assume that processes operate on members of sets. Gradual processes aim at doing something with a particular *range* of items: going – at traversing a distance; writing down – at writing down a series of numbers; assembling – at putting together all parts etc. In general, it seems that for any *result* of a process *F* there exists exactly one set R such that when all members of R have undergone *F*, the result is reached. I will thus say that a process *F* aims at *F-ing* R. To say that a process 'aims at something' is just a way to pick out the process and it means no more than that this process has a specifiable result. Let T be the set of moments. We might now define two notions:

α) completion of a process on a particular date (completion *in* time).
A process *F* aiming at *F-ing* R is α-completed $\equiv_{def}$ there is $t_n \in$ T s.t. R exist at t_n and each member of R has undergone *F*.
β) tenseless completion
A process *F* aiming at *F-ing* R is β-completed $\equiv_{def}$ the set of items which have undergone *F* at *any* time = R

Here, R itself does not need to exist at any particular time. That R is completed means that if we take the set theoretical union *over* times, of sets of items processed in those times, the result will be R. We can thus express the difference between α- and β-completion in set-theoretic terms. Let R* be the set of sets of *F*-processed items. Let
f: T $\rightarrow$ R* be a function representing the actual process *F* (that is, f tells us how much has already been done at any particular time). Then:
F is α-completed $\equiv \exists t_n \in T\ (f(t_n) = R)$
F is β-completed $\equiv \bigcup_{t \in T} f(t) = R$
Any process which is α-completed, is thus also β-completed, but not conversely.

Now, it seems clear to me, that in ordinary language when we talk about processes being complete we mean α-completion. We say that a process was, or can be, or will be completed *in* 10 months, or *in* 20 steps, or *in* 20 moves (if we take a game of chess for example). It seems to me that when someone says that *F* is done *in* some time, she sets a *time-limit*, *after* which the process should have been completed. The English language has even a separate grammatical tense to express predictions about completion of processes – Future Perfect. But can we say that a process will have been completed *in* infinite time, or *after* infinitely many steps? If we think about the future, this is nonsensical: there is no future *after* infinitely many steps, to contrast with the earlier future – infinity takes up the whole future.[4] Consider next the following point: if someone will be *always* doing something, he will *never* finish doing it. This seems to me quite analytical. At least in cases like Tristram Shandy's, we spontaneously say, like Smith does, that Tristram Shandy will never finish.

In any case, it is true that Shandy's work will never be completed *for him*. It might be contemplated as a complete whole only by an observer who is *outside time*, like God. But from within time the collection could never be grasped in its entirety, because it does not exist in entirety *in time*, that is, at any particular moment. Grasping this point is of paramount importance for my argument, for it will prevent possible accusations of begging the question. Or so I hope.

Henceforth, when talking of completion, or completion in time, I will mean α-completion.

III. Mathematical representation of gradual processes

Now that it is clearer what is meant by completion, I can take up the issue of successive addition and similar processes. Although some authors, like Smith (1995, p. 88f.), treat it as if it were a matter of psychology or of the phenomenology of mathematical thought, the problem of „successive addition" belongs primarily to mathematics itself.

How can we represent mathematically gradual processes like successive addition? Here is the simplest method:

(1) Take an ordered set equivalent to ω: the set of natural numbers, or of integers, or the set of time intervals, call it T.
(2) We shall call each member of this set a „step".
(3) Take a function *f*, which assigns to each step a set or a sequence, in such manner that the value for each successive step is obtained by a mathematical transformation of the value for the previous step.

[4] I talk about the future to avoid begging the question against those who maintain that there is time *after* infinite past.

(4) At some step, the value of f should be identical to the set which represents the purported result (the process will be thereby α-completed).[5] After that, the value of f should be constant.

Now consider processes of *forming* sets or sequences. The idea is that we take a function which assigns progressively larger subsets of X to successive intervals. At some point X itself should be reached. Let then T be a set satisfying the condition (1) above. Let members of T be indexed by integers in such way that successive numbers are assigned to successive moments. 'P(X)' refers to the power set of X. We might now give a schematic definition:

(SD) Set/sequence X might be formed by a gradual process $\equiv_{def}$
There is an f: $T \rightarrow P(X)$ such that:
(a) f meets the condition specified in (3)
(b) there is $t_n \in T$ s.t. $f(t_n) = X$.
(c) there is $t_k < t_n$ s.t. $f(t_k) \neq X$

Further conditions need to be added in point (c), depending on what sort of process we define, in order to exclude unintended interpretations (that is, when X is somehow almost given and the contribution of the process is insignificant). Point (b) guarantees that X will be formed *in* time, or *in* a particular number of steps – that the process of formation will be α-completed.

Now we can try to define successive addition. The set theoretic difference is symbolized by '\', equivalence by '$\cong$'.

(Df1) Set/sequence X might be formed by successive addition $\equiv_{def}$
There is an f: $T \rightarrow P(X)$ s.t.:
1) for any t_i, t_{i+1}: $(f(t_i) \neq X \Rightarrow f(t_{i+1}) \backslash f(t_i) \cong \{\varnothing\})$ & $(f(t_i) = X \Rightarrow f(t_{i+1}) = f(t_i))$
2) there is $t_n \in T$ s.t. $f(t_n) = X$
3) there is $t_k < t_n$ s.t. $f(t_k)$ is neither X nor any subset of X equivalent to X

By the first condition we obtain a sequence of sets where subsequent sets contain exactly one element more (and after the set is formed the function remains constant). It is the third condition which is truly problematic. If we consider the problem of forming the series of past events by successive addition, it seems that we cannot avoid begging the question either way. For according to the „infinitist", before any given moment, already *infinitely* many events had happened. Hence their values would be sets equivalent to the whole series, and that is forbidden by our definition. Yet if we

[5] Note that although I set the upper limit to a process, I say nothing about the start.

relax the condition, a „finitist" might protest, that if infinity is already in a surreptitious way *given*, the whole problem of *getting* to infinity is begged. However, arguing for the finitist conclusion, I should not beg the question in my favour, so for the time being I will adopt the lax definition:

(Df2) Set/sequence X might be formed by successive addition $\equiv_{def}$
There is an f: T $\rightarrow$ P(X) s.t.:
1) for any t_i, t_{i+1}: $(f(t_i) \neq X \Rightarrow f(t_{i+1}) \backslash f(t_i) \cong \{\varnothing\})$ & $(f(t_i) = X \Rightarrow f(t_{i+1}) = f(t_i))$
2) there is $t_n \in T$ s.t. $f(t_n) = X$
3) there is $t_k < t_n$ s.t. $f(t_k) \neq X$

Now the infinitist might safely claim that the temporal series has been formed by a process which can be represented in this way. We have thus reached a non-question-begging definition of successive addition.

If we think now of *traversing* the past, it is quite natural to represent it by successive subtraction rather than addition. Imagine a distance or series is to be traversed. There has to be some step at which we have not yet reached the end. Then at some step we reach the end; there is no untraversed segment to traverse. Now, we can represent the whole distance by a set, and consequent steps by progressively diminishing sets representing the shrinking distance lying ahead. If the whole distance can be traversed, then the set representing the distance can be *exhausted* by successive subtraction.

Let 'T*' refer to the set of all temporal intervals, which are past or simultaneous relative to some t_0. With each successive moment, the set of all moments traversed up to this step approaches T*, while the distance to t_0 shrinks. We can thus assign to each past moment the distance from t_0: $f(t_i) = T^* \setminus \{\ldots t_i\}$. Thereby we obtain the series:

... $\{t_{-3}, t_{-2}, t_{-1}, t_0\}$, $\{t_{-2}, t_{-1}, t_0\}$, $\{t_{-1}, t_0\}$, $\{t_0\}$, $\varnothing$ which corresponds to the series
...t_{-4}, t_{-3}, t_{-2}, t_{-1}, t_0.

The definition of successive subtraction has the same structure as the definition of successive addition:

(Df3) Set/ sequence X might be exhausted by successive subtraction $\equiv_{def}$
There is an f: T $\rightarrow$ P(X) s.t.:
1) for any t_i, t_{i+1}: $(f(t_i) \neq \varnothing \Rightarrow f(t_i) \backslash f(t_{i+1}) \cong \{\varnothing\})$ & $(f(t_i) = \varnothing \Rightarrow f(t_{i+1}) = f(t_i))$

2) there is $t_n \in T$ s.t. $f(t_n) = \varnothing$
3) there is $t_k < t_n$ s.t. $f(t_k) \neq \varnothing$

IV. Reversibility of addition and subtraction

The crucial idea of my argument is that *if a distance can be traversed in one direction, it should be at least logically possible to traverse it in the opposite direction.* Admittedly, this intuitive formulation encounters obvious problems when applied to time. I believe it logically impossible to travel back in time. But the impossibility derives from the particular nature of the process and has nothing to do with its mathematical properties relevant to our present purposes. Let us then try to interpret mathematically the intuition of 'going in the opposite direction'. We should presumably say that we should 'go' from X to ∅ and from ∅ to X. Thus, if we have defined a function *f* representing 'going' one way, we should be able to define a function which 'starts' at the final t_n of *f* (which marks the completion) and then 'goes back'. The 'direction' in which we moved was represented by the use of indexes in the definition. One method to represent 'moving' in the opposite direction is to simply re-index T in such a way that the final t_n becomes t^0, t_{n-1} becomes t^1 and so on (I use the superscript for the new indexation) . Let *ind* be our basic indexation i.e. a function from T to the set of integers I, which is such that successive numbers are assigned to successive moments. Let t_n be the smallest element of T s.t. $f(t_n) = \varnothing$. Then the new indexation *ind** can be defined as follows:
*ind**: $T \rightarrow I$, such that for all $t \in T$; $n, i \in I$:
(a) if *ind*(t) = *n* then *ind**(t) = 0.
(b) if *ind*(t) = *n* – *i* then *ind**(t) = *i*.

Thus $(t_n) = t^0$ and $(t_{n-i}) = t^i$. Let T* be the set of all temporal intervals, which are past or simultaneous relatively to t_n: $T^* = T \backslash \{t_{n+1},...\}$. Now:

(P5*) If series X might be exhausted by successive subtraction, and *f* is a relevant function, t_n is the smallest element of T s.t. $f(t_n) = \varnothing$, then there is g: $T^* \rightarrow P(X)$ s.t.:
1*) for any t^i , t^{i+1}: $(g(t^i) \neq X \Rightarrow g(t^{i+1}) \backslash\, g(t^1) \cong \{\varnothing\}) \;\&\; (g(t^i) = X \Rightarrow g(t^{i+1}) = g(t^i))$
2*) there is $t^n \in T$ s.t. $g(t^n) = X$
3*) $g(t^0) = \varnothing$

Values of *g* for every moment will exactly correspond to those of *f*. It is also easy to observe that *g* satisfies the definition of successive addition (both in the form of (Df2) and (Df1)). Thus (P3) can be seen to follow from (P5*) as well.

V. The argument and its defence

We are now in a position to construct a proof by contradiction showing that an infinite series of past moments cannot be traversed.

(1) Assume that the infinite series of past moments T* = $\{\ldots t_{-2}, t_{-1}, t_0\}$ has been traversed in one direction. T* is equivalent to ω.
(2) If a series can be traversed, it can be exhausted by successive subtraction. (by P2)
(3) T* can be exhausted by successive subtraction.
(4) There is a function meeting the conditions specified in (Df3).
(5) Let us then take f: T* $\rightarrow$ P(T*) s.t.:
1) for any t_i , t_{i+1}: $(f(t_i) \neq \varnothing \Rightarrow f(t_i) \backslash f(t_{i+1}) \cong \{\varnothing\})$ & $(f(t_i) = \varnothing \Rightarrow f(t_{i+1}) = f(t_i))$
2) $f(t_0) = \varnothing$
3) there is $t_k < t_0$ s.t. $f(t_k) \neq \varnothing$
(6) t_0 is the smallest element of T* s.t. $f(t_0) = \varnothing$. So under the new indexation *ind**,
*ind**$(t_0) = t^0$.
(7) By application of (P5*):
there is g: T* $\rightarrow$ P(T*) s.t.:
1*) for any t^i, t^{i+1}: $(g(t^i) \neq T^* \Rightarrow g(t^{i+1}) \backslash g(t^1) \cong \{\varnothing\})$ & $(g(t^i) = T^* \Rightarrow g(t^{i+1}) = g(t^i))$
2*) there is $t^n \in T$ s.t. $g(t^n) = T^*$
3*) $g(t^0) = \varnothing$
(8) Lemma: there is no function which can meet the conditions specified in (7).
Proof:
(1') Assume g is such function
(2') T* is infinite
(3') Let $g(t^n) = T^*$. Then $g(t^n)$ is infinite.
(4') By induction on indexes of arguments of g we will show that for none of
them the value of g is infinite
Step 0: $g(t^0) = \varnothing$. Therefore $g(t^0)$ is not infinite.
Step 1: Assume $g(t^i)$ is finite. Therefore there is a finite $n \in \omega$ s.t. $n \cong g(t^i)$.
$g(t^{i+1})$ contains exactly one element more than $g(t^i)$.
Therefore $g(t^{i+1}) \cong n+1$

If n+1 were infinite, n would be infinite too (think of an equivalence function from a proper subset of n+1 to n+1. If we restrict the domain to those elements whose values are different from n, we will obtain an equivalence function from a proper subset of n to n).

Therefore $g(t^{i+1})$ is not infinite.

Therefore for no i, $g(t^{i})$ is infinite.

Contradiction (3'), (4').

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Contradiction (7), (8).

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It might be contended that in demanding that in going back T* be reached by successive addition *at some point* I have begged the question after all. For according to the infinitist there is no starting point to which we could go back. She might insist that *in an infinite time* T* could be reached. I think that on the contrary, this shows what is wrong with the conception of traversing an infinite distance. For if it takes infinite time to complete the supposed process of traversing, then evidently it has to be understood as a process which can only be β-completed. Yet it makes no sense to think that the very process of traversing the past, which must reach a particular time – the present – can be completed only in some timeless manner.

Let me try to make this charge a bit clearer. It might help to attend to an interesting tension in Smith's defence of the infinite past. Smith is highly concerned to reject the idea that infinity of the past entails that there are two events separated by an infinite distance (Craig, Smith 1995, pp. 80-83). In this he is certainly right. But why, we may ask, should that cause him any worry in the first place? Suppose that it makes sense to say that an infinite past *has* elapsed. Thus it is true that the process of traversing *an* infinite series *has been completed*. But if traversing an infinite series, which has no beginning, and has an upper boundary, can be completed, then surely it should also be possible to traverse an infinite series that is bounded on both sides? Consider the following case:

My Non-Standard Future

Suppose that I begin to exist but I am immortal. The set of the moments of my life which includes the first moment of my existence and all the moments which succeed it: T = $\{t_0, t_1, t_2...\}$ is thus infinite. But on top of that, God has decided to give me some extra time: E = $\{e_0, e_1, e_2...\}$. The moments in E are ordered just as in T, but each of them stands in the relation

„later than" with respect to *all* moments of T. My life is thus: L = T + E = $\{t_0, t_1, t_2... e_0, e_1, e_2...\}$. Now, it should be observed, that e_0 is not the successor of any moment in T, and, secondly, there are infinitely many moments between t_0 and e_0. The question is: could I really live through the moments in E?

What should Smith say about this case? In keeping with his approach it should be said that traversing T *can be completed*; „in infinite time", to be sure, but completed nevertheless. This means that moments of E will be somehow reached and I will live through them. But this is utterly absurd. It is quite evident that I will never get to them.

It might be retorted that the impossibility stems from the peculiarity of the *ordering* of L; namely, from the fact that e_0, the upper bound of the infinite series $\{t_0, t_1... e_0\}$, is not the successor of any earlier moment in T. In contrast to this, in the set of past moments T* = $\{... t_{-2}, t_{-1}, t_0\}$, all elements, including t_0 are successors of previous moments, and so the present can be reached. But this feature is in fact irrelevant. First, the distance, the size (cardinality) of the series traversed, is the same. Secondly, we can construct a series of past intervals of time in such a way, that the present moment will not be the successor of any previous interval. We can divide the past in one-second intervals, up to the very last second, which we divide into a half of a second, a quarter, and so on. The resulting series T' can be represented in a simple way: ...-3, -2, -1, -½, , -¼... 0. Now, if infinite series, and T* in particular, can be traversed, then surely T' can be traversed as well. The fact that its upper bound is not the successor of any earlier element does not matter.

I conclude that there is no relevant difference between the completion of traversing T and of traversing T* or T'. They should be completable in the same way.

Now, if „completion" implies the actual reaching of the aim (as it should if T* is to be traversed), then T will be traversed, and e_0 reached, which is absurd. So T (and consequently T*) has to be completable in some other sense. But, as we have seen time and again, a processes like traversing T can be completed only in a very queer sense. In fact, the aim of the process *cannot* be reached, as Tristram Shandy's case, and *My Non-Standard Future,* show. The talk of „completion in infinite time" has to be unpacked in the form of some such phrase as: „if the process goes on for a sufficiently long time (i.e. goes on infinitely), then all the elements in the range will undergo the process". The use of tense is crucial: we *cannot* say „all elements (i.e. all of them collectively) will *have* undergone the process"! We cannot use the perfect in this way. But that is precisely what we would have to say if traversing the past were possible: all these moments, the whole set, *have* been traversed. But we are not allowed to say that. Furthermore, in the case of traversing time itself, the explanatory phrase for „completion" is altogether unilluminating; it comes down to saying „if one traverses time for a sufficiently long time (infinitely), infinitely many moments will be traversed". Now, if this means that an infinite series of moments (if it were possible) could be mapped onto itself, that's quite true, but this simply does not show in any way that the end could be *reached*. On the other hand, we could try to map T* onto another series, onto some static super-time. It is by means of such procedure that one can imagine a process like writing Tristram Shandy's works to be completed, *all* the books as existing and given all at once. But this is a view from *outside* time. It does not show what can exist and be completed *in* time.

To sum up – the idea of traversing an infinite series is inconsistent with intuitive principles which express our understanding of traversing a distance. Secondly, no clear sense can be given to the notion of *completing* the traversing of an infinite time-series.
These are the strongest possible reasons one can have to reject the idea.

VI. Philosophical interest of the argument

I would like to finish my paper by pointing out some philosophical implications of my argument and its conclusions. First of all, they could contribute to an argument for the existence of God. I think, however, that the fact that time, and so the universe too, has a beginning does not guarantee that they need an external cause. It does seem necessary if we accept the A-theory of time: a thing popping up into existence without any reason is not something we should accept. But if time were a dimension of the universe which just is out there – and this 'is' cannot be further tensed – then the need to posit a cause seems less nagging.

The second application concerns our life. Even if time itself does not „flow", we are certainly moving in time. Our argument has shown that our journey necessarily has a beginning. It is interesting to note that almost all great schools of Indian philosophy made a contrary assumption (Potter 1963). The process of reincarnation was conceived as having no beginning in time. But this has been shown to be impossible. Our conclusions have also a bearing on the problem of the asymmetry of our attitudes to *ante natam* and *post mortem* non-existence. If it is inconceivable that we could live for an infinite time before our birth, then we might be right in not being concerned about it all. Yet it is *prima facie* conceivable that our life has no end. So our concern about the time after our life's end, but not the time before its beginning, might prove rational.

References

Craig, W.L., 1979, *The Kalam cosmological argument*, Macmillan, London.

Craig, W.L., Smith Q., 1995, *Theism, atheism, and big bang cosmology*, Clarendon Press, Oxford.

Oaklander L.N., 1994, „On the Experience of Tenseless Time", in Oaklander L.N., Smith Q. (eds), *The New Theory of Time*, New Haven: Yale University Press , pp. 344-350.

Oaklander L.N., Smith Q. (eds), 1994, *The New Theory of Time*, New Haven: Yale University Press.

Potter K.H., 1963, *Presuppositions of India's Philosophies*, Englewood Cliffs: Prentice-Hall Inc.

Russell, B., 1993, *Our Knowledge of the External World*, London: Routledge.

Russell, B., 1937, *The Principles of Mathematics*, second edition, London: Allen and Unwin.

Small, R., 1986, „Tristram Shandy's Last Page", *The British Journal for the Philosophy of Science* 37, pp. 213-216.

Williams, C., 1994, „The Phenomenology of B-Time", in Oaklander L.N., Smith Q. (eds), *The New Theory of Time*, New Haven: Yale University Press, pp. 360-372.