TRACTARIAN ONTOLOGY: MEREOLOGY OR SET THEORY?

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Abstract. I analyze the relations of constituency or „being in" that connect different ontological items in Wittgenstein's *Tractatus Logico-Philosophicus*. A state of affairs is constituted by atoms, atoms are in a state of affairs. Atoms are also in an atomic fact. Moreover, the world is the totality of facts, thus it is in some sense made of facts. Many other kinds of Tractarian notions – such as molecular facts, logical space, reality – seem to be involved in constituency relations. How should these relations be conceived? And how is it possible to formalize them in a convincing way? I draw a comparison between two ways of conceiving and formalizing these relations: through sets and through mereological sums. The comparison shows that the conceptual machinery of set theory is apter to conceive and formalize Tractarian constituency notions than the mereological one.

An observation by Gottlob Frege on the Tractarian ontology is an excellent introduction to our topic. The observation is included in a letter written by Frege to Wittgenstein, where Frege makes some critical remarks on the ontological sections of *Tractatus*.[1] In this context, Frege writes:

> It is essential for the thing that it can be a component of an object-state. Can now a thing be also a component of a fact? The part of the part is part of the whole. If a thing is component of a fact and every fact is part of the world, then the thing is also part of the world.

Frege tries to apply the mereological notions of part and whole to the Tractarian ontology. He suggests that this application risks making the Tractar-

[1] Frege's existing letters to Wittgenstein have been published in Frege 1989. The quoted translation by R. H. Schmitt appeared in Frege 2003.

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ian ontology inconsistent. Frege's argument can be summarized in the following steps:

(1) the notion of part is obviously transitive (*general truth*);
(2) the Tractarian notion of constituency should be understood as the mereological relation of „being part of" (*exegetical assumption*);
(3) an atomic fact is constituted of objects (*exegetical assumption*);
(4) the world is constituted of atomic facts (*Tractarian explicit thesis*);
(5) the world is constituted of objects (*logical consequence of 1), 3) and 4)*);
(6) the world is not constituted of things, *i.e.* objects (*Tractarian explicit thesis*).

(5) and (6) are openly contradictory. Therefore, if one is disinclined to concede that *Tractatus* is inconsistent, at least one of the two exegetical assumptions (2) and (3) has to be dropped. Either it is not true that facts are constituted of objects, or it is not true that the Tractarian notion of constituency is conceivable as the mereological notion of 'being part of'.

Frege – in the quoted excerpt – seems to be in doubt about (3). On the contrary, I am going to propose that (2) should be dropped, since it is an untenable way of conceiving the Tractarian ontology and an ineffective way of formalizing it. The mereological conception of the objectification of classes is incompatible with some explicit core theses of *Tractatus*. Frege's objection will turn out to be an example of this incompatibility: the elementary, undefinable dyadic relation between objects in mereology is transitive and we can not use a transitive relation in order to explicate this well-known Tractarian core – thesis:

1.1 The world is the totality of facts, not of things (Wittgenstein 1961b).

On the contrary, the undefinable dyadic relation 'being an element of' in set theory is not transitive. Thus, it seems at a first glance a better candidate for conceiving and formalizing the Tractarian relations of constituency.

First of all, I need to explain briefly why it seems to me implausible to block Frege's argument denying the exegetical assumption (3). Why should we be confident that atomic facts (molecular facts will be discussed later) are constituted of objects? Could not we reject this assumption and gain more ease in dealing with constituency relations in *Tractatus*? Actually, this objection is an instance of a general problem which the approach of this paper can raise.

Each ontological notion involved in constituency relations in the Tractarian ontology has been analyzed in depth in the wide exegetical literature concerning *Tractatus*. It would not be easy to discuss – nor merely to list –

the several different readings about the nature of atoms, states of affairs, atomic facts, molecular facts, world and logical space. However, we can legitimately ignore most such debates: we are not interested in the role of these ontological items in the semantics and epistemology of *Tractatus*, nor in the actual identification of these items in any kind of model or external reality. We aim to focus exclusively on the constituency relations and many exegetical issues about the nature of what is constituted and what constitutes have no clear bearing on this issue.

On the other hand, there are other exegetical debates we are forced to deal with. In particular, we need to decide if *Tractatus* actually maintains principles such as (3) and (4) in Frege's argument. The typical form of such principles is, for ontological types x and y: „(for any x) an x is constituted of (a certain number) of ys".

Why are these principles relevant? Frege's argument offers a clear example: if the constituency relations are conceived or formalized in a certain way, then the subsistence of some constituency relations implies the subsistence of certain other constituency relations; the subsistence of these other constituency relations can be an unwelcome consequence, since, e.g., it is explicitly denied by *Tractatus*.

In other cases we will analyze below, further problems can stem from an extensionalist approach to the Tractarian ontology as a whole. According to this extensionalist approach, we can give adequate identity conditions for the complex entities mentioning only their simpler constituents. The viability of this approach, while not being required by most of the arguments developed, is important for the relevance of our analysis and will be therefore briefly discussed – with special reference to states of affairs and facts – in the last part of the paper.

Leaving aside the problem of extensionalism, we have to decide first of all which constituency relations obtain in the Tractarian ontology. Although these decisions are less controversial than those concerning the nature of the ontological items in themselves, there is nothing like a general agreement.

I can not provide here a discussion of these problems. We simply rely on the prevailing exegesis. It is possible to verify that these readings are actually prevailing, resorting to reference works on Wittgenstein terminology. E.g., in its main entries concerning the ontological items of *Tractatus*, inclines towards the positions I will rely on in this paper, while confronting them with alternative readings. Here I can only make explicit my consequent options and quote the Tractarian sections supporting this conclusion, ignoring more controversial textual evidence.

I. States of affairs and atomic facts are constituted of objects

Concerning states of affairs, there is a quite explicit Tractarian section:

> 2.011 It is essential to the thing to be a possible constitutive part of a state of affairs.

It seems clear, then, that states of affairs are constituted of objects. However, it is not fully clear what a state of affairs (translation of the German *Sachverhalt*) is: there are two readings of this term and the prevailing exegesis is that Wittgenstein employs the term in two different senses, in different Tractarian sections. In the first sense – confirmed by Wittgenstein himself in a later letter to Russell (Wittgenstein 1961b, p. 125) and embodied in the Ogden-Ramsey translation (Wittgenstein 1922) – *Sachverhalten* are nothing else than atomic facts. On this reading, atomic facts are constituted of objects.

However, Wittgenstein, at least in some places, employs *Sachverhalt* with a different sense: in these contexts, a state of affairs is a possible atomic fact: *i.e.* what is pictured by an atomic proposition independently of its truth-value. In particular, a false proposition pictures a state of affairs, but this state of affairs does not subsist, *i.e.* it is not a fact. Throughout this paper– following a common exegetical practice – we use 'state of affairs' in this second meaning (the other meaning is expressed by 'atomic fact').

Anyway, in the prevailing analysis of these notions, atomic facts are constituted of objects also when they are not identified with *Sachverhalten* in the section quoted above. In fact, the class of atomic facts is a subclass of the class of states of affairs: a state of affairs is an atomic fact iff it subsists. The equivalence of „atomic fact" and „subsisting states of affairs" has been shown through the comparison of many Tractarian sections. Consider, for example, the following couple of sections:

> 1.1 The world is the totality of facts. [...]
> 2.04 The totality of existing states of affairs is the world.

From this point of view, if all the states of affairs are constituted of objects, then the subsisting states of affairs, in particular, are also constituted of objects. Thus, atomic facts are constituted of objects: premise (3) of Frege's argument is sound, in spite of Frege's skepticism.

It should be noted that – in the present context – we are not implying that states of affairs and atomic facts have no other constituent than simple objects. We are simply saying that atomic objects are among their constitu-

ents. However, the stronger thesis is deeply connected with the general viability of an extensionalist approach to the Tractarian ontology and will be discussed briefly in the last part of this paper.

II. The world is constituted of objects

The subsistence of this constituency relation is expressed with heterogeneous locutions, in the lexical field of constituency.[2]

1.1 The world is the totality of facts. [...]
1.2 The world is divided in facts.

Thus, we can accept the fourth premise of Frege's argument.

III. Logical space is constituted of possible states of affairs

Tractatus, while often mentioning the notion of „logical space", does not provide an explicit definition for it. What is not defined is the logical space in general. On the contrary, there is a definition of the logical space of an object: the logical space of an object is constituted by the states of affairs which have that object as a constituent:

2.013 Any thing is, as it were, in a space of possible states of affairs. [...]

Because of this definition and of the role played by the notion of logical space in the Tractarian ontology, the logical space *tout court* is often seen as the union of the logical spaces of the single objects, that is as the class of all the states of affairs.

We now have all the data needed in order to face our main topic. I derived from Frege's letter an argument to maintain that mereology is an unreliable tool in conceiving and formalizing the Tractarian constituency relations.

[2] It should be parenthetically noted that in this paper we do not provide an analysis of this heterogeneous lexicon: words such as „totality" (*Gesamtheit*), 'constitutive part' (*Bestandteil*), 'divide in' (*zerfallen in*) are used with apparent freedom and this freedom discourages any confidence in the eventual historical and theoretical connotations of these words. This does not exclude that, in specific contexts, the resort to a certain expression instead of another may have a philosophical significance. These aspects are not covered here.

Such an argument is built on the need for the non-transitivity of a binary relation. The argument could seem weak, in particular from the point of view of formalization. Why should we need a *primitive* binary relation? Can not we introduce by definition a non-transitive binary relation, in order to formalize the constituency relations? It is very easy to introduce such a non-transitive relation also in the mereological framework.[3]

Other objections can be raised against the choice of set theory as a plausible alternative. One could be worried about Wittgenstein's general perplexity with regard to set theory: a well-known theme, which is common to all the different periods of Wittgenstein's philosophical work. In particular, the Tractarian philosophy of mathematics has been described as a 'logicism without classes' (Frascolla 1994); thus it could seem hazardous to reintroduce classes – which Wittgenstein excludes from their most straightforward philosophical context – in the deepest articulations of the Tractarian ontology.

In the remaining part of this paper, I try to develop positive arguments for the choice of set theory as a pertinent tool and negative arguments against the objections summarized above. As a first step, I propose that Wittgenstein's perplexity concerning set theory does not concern us in this context. This first step is mostly methodological, since it concerns the general meaning of my analysis of the Tractarian constituency relations and of any formalization of the Tractarian ontology as a whole.

First of all, we should not overestimate the historical pertinence of the distinction between set theory and mereology. As we have seen, Frege was aware that the use of relations such as 'being part of' and 'being constituted by' has certain systematic consequences which deserve serious consideration. However, there is no evidence that Frege considered the mereological framework as an alternative to a set theoretical one, in the general problem of the representation of classes. Concerning Wittgenstein, there is no trace of the direct application of notions as class or element to ontology in general, nor to his own ontology in *Tractatus*.

Consistently, Wittgenstein thought that the pertinent field for the notion of class is not ontology, but philosophy of mathematics: in this field, he

[3] E.g. the relation of 'not being a part of' is not transitive, though it is not an intuitively convincing counterpart for the Tractarian constituency relations. Otherwise, if the relation of 'being part of' is a total order of the domain of objects, then we can easily define the relation of 'being an immediate part of': an object a is immediate part of an object b, iff a is part of b and there is no object c such that a is part of c and c is part of b.

shows much perplexity and tries in *Tractatus* to avoid any resort to the notion of class. But the Tractarian philosophy of mathematics has no perspicuous connection with the Tractarian ontology: since the definition of natural number relies on the indefinite iterability of logical operations, mathematics is founded on those aspects of logic, which concern molecular propositions as truth functions of atomic propositions.

Thus, in the conception and formalization of the relations between objects and states of affairs, between facts and world, and so on, we ought not to be influenced by these themes. We should face directly and from the most abstract point of view the general problems of the relations *one-many many-one*. In order to accomplish this task, we need to make more explicit and slightly more formal the distinction between the set theoretic and the mereological approach to the problems of ontological constituency.

Let the notion of class be intuitive and not problematic. As a definition, given a certain property, there is the class of objects with that property. The problems arise when we want to objectify these classes: that is to say, we want to treat them – directly or indirectly (i.e. through an object representing the class) – as objects ontologically homogeneous with their members, so that, for example, we can quantify over a mixed field of members and classes.[4]

[4] A first objection concerning *Tractatus* can be raised: why should we deal with such quantifications from a Tractarian point of view? Though the Tractarian theory of quantification is exegetically debatable, it seems clear that the field of quantification is given by the *denotata* of the names which can be substituted to the quantified variable obtaining a well-formed proposition. Since the name of an atom can be substituted only by the names of certain other atoms, we never need to quantify on a mixed field of entities. Moreover, it is possible to deal with the problem of treating classes as objects without the pretense of ontological homogeneity. The theory of types explored this possibility, and *Tractatus* seems explicitly committed to some form of theory of types, meant as a chapter of a wider logical syntax (cf. *Tractatus* 3.331-3.334). In such a picture, why should one worry about ontological homogeneity?

This objection allows us to clarify what I mean by 'formalization' of the Tractarian ontology. We do not aim to formalize the Tractarian atomic propositions. On the contrary, I aim to formalize philosophical-metaphysical claims, such as „the world is constituted of facts", which *Tractatus* itself declares to be inexpressible in the meaningful language, as *Tractatus* conceives it. Thus, we do not need to cope with the Tractarian theory of quantification: my formalizations need not to be part of the Tractarian meaningful language. Furthermore, the ontological theses we want to formalize actually require us to quantify on all the items admitted by the ontology, without any distinction of type. E.g., when we formalize the thesis that „the world is the totality of facts", we need to quantify on all the objects of the ontology, in order to express that all the objects which are facts – and no other object – are members of the world (class of facts).

When a class can not be treated directly as a class, but needs to be objectified by a second object, we can say that this object represents a class. We can focus on this relation of representation, (when classes are regarded as objects themselves, this relation of representation will be a subclass of identity). There are two main problems concerning the relation of representation:

1. to determine the field of the relation of representation – say $O(y,X)$ – where y is the representing object and X is the represented class;
2. to understand the nature of the relation $O(x,Y)$, i.e. to clarify what we mean when we say that y represents X.

Concerning *Tractatus*, the first task can be reduced to the determination of the range of classes we need to represent.[5] Besides simple objects, states of affairs, atomic facts, world, logical space, how many other classes do we need to consider? There are many different conceivable entities, about which *Tractatus* is silent or unclear. We discuss two kinds of them, which turn out to be interesting for different reasons.

IV. Ill-formed groups of atoms

These are the classes of atoms which are not states of affairs, being for some reasons ill-formed. When we define which groups of objects are states of affairs or when in general we need to quantify on all the possible groups of atoms in order to identify those groups with a specific feature, we actually require that any group of atoms be treatable as an entity. Although *Tractatus* never talks of such ill-formed n-tuples of atoms, we need, nevertheless, to be able to treat them as objects.

This is an exemplification of a general issue: it is not enough to deal with metaphysically salient entities, such as states of affairs, world and so on. We should rather justify the choice of a language and of a conceptual apparatus for the definition of such salient entities. But in these definitions we need to quantify also on non-salient entities.

[5] It is a reduction, since: we can ignore the hypothesis that a class is represented by more than one object; we can avoid facing some classically ticklish classes – the empty class *in primis* – which are clearly irrelevant in my analysis.

V. Molecular facts and situations

According to Peter Simons (Simons 1986), it is interesting to determine if molecular propositions correspond to some entities in the Tractarian ontology and which kind of entities these are. In fact, it seems that the Tractarian term 'fact' does not denote only the atomic facts, but also more articulate facts, and these molecular facts could be the truth-makers of molecular propositions. Moreover, the term 'situation' (*Sachlage*) would denote something which is for molecular facts what states of affairs are for atomic facts: they would correspond to molecular propositions, independently of their truth value. Thus, a systematization of the Tractarian ontology should find a place for both these kinds of entities.

Simons, in the essay we are referring to, uses the formal and conceptual apparatus of mereology. However, the problems he has to face with the notion of „situation" would be quite similar also with a set theoretical apparatus. The problem is that, except for the specific case of a simple conjunction, there is no fixed group of states of affairs corresponding to a molecular proposition. For example, in the case of a simple disjunction „$\alpha \vee \beta$", can we identify the situation which, if it subsisted, would make true the disjunction? The disjunction is true in three cases. Given such a variety of truth conditions, how is it possible to find a unique entity, which, subsisting, makes true the molecular proposition.

Simons tries to resort to the notion of *ens successivum*, introduced in the mereological *milieu* by Roderick Chisholm (Chisholm 1976), in order to cope with some classical philosophical problems affecting entities with variable identity conditions. This solution would deserve to be considered in depth. However, it is unable to deal – as Simons himself admits – with the semantics of quite important molecular propositions, such as simple negations.[6]

[6] That means that no *ens successivum* can correspond to a molecular proposition resulting from the application of the logical operation of negation to an atomic proposition. One could be tempted to resort to the difficult Tractarian notion of negative fact , but it seems that no plausible conception of a negative fact is suitable for the role of a „situation", as Simons conceives situations. A situation should be something which subsists iff the corresponding molecular proposition is true: but what would it mean for a negative fact to subsist? *Tractatus* defines a negative fact as the non-subsistence of a state of affairs (cf. *Tractatus* 2.06), but what would it mean for a non-subsistence or – according to the most straightforward exegesis – a non-subsistent entity to subsist? Thus, the notion of *ens suc-*

Given these difficulties, Simons' general aims should perhaps be disputed. Does the Tractarian ontology really maintain that there are significant ontological counterparts of molecular propositions? Do the occurrences of the technical terms 'situation' and 'fact' really imply that molecular propositions have an ontological counterpart? A traditional reading of *Tractatus*[7] maintains that world and language are connected at the level of names and atomic propositions, while molecular propositions are only truth functions of the atomic propositions constituting them. The molecular propositions – as the atomic ones – are intrinsically affirmed, since they say that things stand as they show (*Tractatus* 4.022). Therefore, they say that one or another of a certain group of combinations of the subsistence and non-subsistence of states of affairs subsists. Thus, it seems that no other entity besides states of affairs is involved in their semantic content. If this reading of the Tractarian theory of molecular propositions is sound, then we do not have any systematic reason to become entangled with *entia successiva*, nor with any possible replacement of them.

In spite of this lack of systematic reasons, some textual evidences could be quoted. However, I maintain that none of these occurrences of *Tatsache* (the German word for fact) or *Sachlage* (the German word translated with situation) implies a straightforward connection with the semantics of molecular propositions.

We can not provide here a detailed analysis of the occurrences of these terms in *Tractatus* (Glock 1996). About *Sachlage*, it is enough to note that Simons' reading of this term is almost unique in the exegetical literature: sometimes this term is not considered as a technical term with a constant meaning, sometimes it is read in connection with its wider use in *Notebooks 1914-1916*, where it seems to be coreferential with the Tractarian *Sachverhalt*. Anyway, a massive and problematic enrichment of the Tractarian ontology is not justifiable on the basis of such a semantically shifting term.

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cessivum seems to need at least some modifications. One could for example maintain that the *ens successivum* corresponding to a certain molecular proposition is not constituted by the states of affairs pictured in the atomic propositions constituting the molecular one, but by the possible worlds where the molecular proposition turns out to be true. However, this further proposal leads us far from my main goal: the ontological status of intermediate entities between states of affairs and atomic facts on one side and the logical space and the world on the other. The revised *entia successiva* would be more complex than the world, and would include a certain number of subsets of the logical space.

[7] Frascolla 2007 sets forth old and new arguments for this traditional reading.

On the contrary, we need to analyze some occurrences of '*Tatsache*', since they refer certainly to a kind of entities we need to represent. But these entities – which can be safely called molecular facts – do not hold any direct relationship with the semantics of molecular propositions; moreover, when these entities are analyzed from a non-semantical point of view, they are much less ticklish.

It is undeniable that some Tractarian occurrences of 'facts' can not designate only subsisting states of affairs, *i.e.* atomic facts. The following is perhaps the least controversial of the quotable excerpts:

> T 2.034 The structure of a fact [*Tatsache*] consists of the structures of states of affairs [*Sachverhalten*].

A singular 'fact' is said to consist of a plurality of 'states of affairs': it can be considered a molecular fact.

From an ontological point of view, the easiest and most plausible solution is to conceive them as classes of atomic facts, those atomic facts whose structures constitute its structure. The only context where this solution could be unsatisfying is the semantics of molecular propositions, since only in some very specific cases is there a class of atomic facts which is suitable to be the semantic value (in this context, in the sense of the semantic truth-maker) of molecular propositions: only a conjunction of atomic propositions is true iff all the atomic facts in a certain class (this class would be the molecular fact) subsist. No other kind of molecular proposition is such that we can circumscribe a unique class of states of affairs whose subsistence makes true the molecular proposition.

But if the above mentioned traditional reading of the Tractarian theory of the molecular propositions is sound, then the straightforward identification of molecular facts with classes of atomic facts is fully satisfying. The truth-makers of a molecular proposition are the truth values of the atomic propositions constituting it: Tractarian semantics does not need any other entity, and in particular no medium-size entity between atomic facts and the world. However, *Tractatus* undeniably mentions such medium-size entities. It is legitimate to try to find a connection between such entities and the Tractarian semantics of molecular propositions, since otherwise it is not possible to understand why *Tractatus* mentions these entities at all. But, in front of the unsatisfying results, one should note that this connection between molecular facts and the semantics of molecular propositions is neither explicitly drawn nor even merely mentioned in *Tractatus*. More-

over, *Tractatus* has a well-developed semantics for molecular propositions and this semantics does not mention molecular facts at all[8].

Therefore, we can safely choose to conceive the molecular facts mentioned in *Tractatus* as unproblematic classes of atomic facts, ignoring any connection with the semantics of molecular propositions.

This review of two kinds of non-standard entities will be useful in order to choose between a mereological and a set theoretical approach in conceiving and formalizing classes of entities in the Tractarian ontology. The second kind of entity reviewed is relevant in order to exclude our having to deal with something more complex than mere relations of constituency between entities and classes of them: molecular facts do not require that my apparatus be able to handle problematic entities such as *entia successiva*.

We can now answer the second question concerning the relation of representation. What does the locution $O(y,X)$ mean exactly? X is a class, with certain members; on the other side of the relation O, what is y and what does y contain? One feature of y is common to the different understandings of the representation of classes: a representation of X needs to include at least all the members of the class X. In order to choose a provisional neutral term, we can say that y is an aggregate and we can call components the entities that an aggregate contains.

Let then η be the relation of being a member of a class and $\prec$ the relation of being a component of an aggregate. The generally accepted principle can be formalized as follows:

$$(1) \qquad O(y, X) \rightarrow \forall z(z\eta X \rightarrow z \prec y)$$

The implication in the opposite sense is, on the contrary, highly problematic: has the representation of X other components than the members of X and, if so, which other components? The easiest answer is to deny the need for other components. According to this view, the aggregate representing a class has as its components all the members of the class and nothing else. An advantage is that we obtain easily a necessary and sufficient condition for a certain aggregate to be the representation of a certain class.

$$(2) \qquad O(y, X) \leftrightarrow \forall z(z\eta X \leftrightarrow z \prec y)$$

[8] Cf. *Tractatus* 4.41. Truth-possibilities of elementary propositions are the conditions of the truth and falsity of propositions.

In this reading we can say that y is a set and has as its elements all the members of X and nothing else. This is the set theoretic conception of representation and $z \varepsilon y$ is much more usual than $z \prec y$.

We can immediately try to apply to *Tractatus* this point of view: a state of affairs will be represented by a set whose elements are the atoms constituting the state of affairs. The subsisting state of affairs – i.e. an atomic fact – will be represented by that same set of atoms. The logical space will be represented by the set of all the states of affairs. The world will be represented by a subset – potentially empty and potentially improper[9] – of the set of all the states of affairs.

The formula (2) allows us also to narrow our store of notions: we can say that, from the set theoretic point of view, the relation $O(x,Y)$ is the identity: represented classes and representing sets contain just the same components, so it is much easier to identify classes and sets.[10]

On the contrary, a different historical tradition admits that the representing aggregate can contain something more than the members of the represented class. According to this tradition, the representing aggregate includes also, as components, the components of the members of the represented class, the components of these components, and so on. Furthermore, it also includes as components all the objects obtained by aggregating in any way all the other components.

An example can clarify this point of view and show its intuitive motivations. Consider the class of points in a given square. This class is not represented by an entity including only those points: the suitable entity includes also the diagonals of the square and in general any segment included in the square; but also the inscribed triangles, and so on. This representing object should be – so to say – the square itself and, as such, is the representation not only of the class of the points in the square, but of other classes too: for

[9] These two border-line eventualities are a straightforward consequence of the so-called 'postulate of independence' for states of affairs. The possible emptiness of the logical space can be also derived from the fact that the logical space in general is the set theoretic union (or the mereological sum) of the logical spaces of each single atom: and the logical space of atoms can be empty, as *Tractatus* explicitly maintains in section 2.013. However, some scholars maintains that the logical space can not be empty, see for example Gale 1976.

[10] We are not implying that each possible class (i.e., each class defined by a property) is a set, i.e. an object, since this would lead to the paradoxes following an indiscriminate validity of the axiom of comprehension. We are maintaining merely that each representable class is a set, an object. Any relevant entity in the Tractarian ontology can be an object without carrying paradoxical consequences.

example, of the class of all the plane figures included in it, and also of any class of two pieces resulting from a partition of the square.

These intuitions cannot be immediately translated into a necessary and sufficient condition for a certain aggregate to represent a certain class. In the absence of an equivalent of (2), we need to regulate the general behaviour of $\prec$ by means of specific principles. In other words, since the representing object includes something more than the members of the represented class, then we need to determine what it does actually include, meeting the intuitions expounded about the square.

Therefore, we have to include two kinds of components: the components of the members of the class have to be components of the representing aggregate; any union of components of the aggregate has to be a component itself. The cardinality of the aggregate risks exploding, thus we need a minimality condition.

We can formalize these laws in the following three formulas. The first and the second formula affirm that the relation $\prec$ is reflexive and transitive. The third one actually identifies the representing object, with reference to a plausible minimality condition.

$$x \prec x$$

$$x \prec y \wedge y \prec z \rightarrow x \prec z$$

$$O(y, Y) \leftrightarrow \forall z(z\eta X \rightarrow z \prec y) \wedge \forall u(\forall z(z\eta X \rightarrow z \prec u) \rightarrow y \prec u$$

This conception of the representation of classes can be called – with a certain degree of simplification[11] – mereological.

We can now try to apply this point of view to the Tractarian ontology. Let us consider first the states of affairs, which are classes of atoms. According to the principles stated above, the representing object will not include only the atoms. It should also include any union of the atoms: any non-atomic part of a state of affairs, a kind of entity which does not seem salient at all in relation to the Tractarian ontology. Therefore, the need of

[11] The simplification is both historical and theoretical. In fact all the principles stated above are shared by a different conception of the relations one-many many-one, which is usually labelled as Boolean. Because of these similarities, the mereological and the Boolean approach to these problems receive sometimes the joint label of 'summative conception'. Our specific aims allow us to ignore the differences between the mereological and the Boolean approaches.

including them in a state of affairs is a first unwelcome consequence of the mereological approach.

As a second test, let us consider a particular species of the deviant n-tuples of objects considered at the point 1) of the above review: the n-tuple including all the atoms constituting two states of affairs and nothing else. These n-tuples do not constitute a salient entity in the Tractarian ontology and we should avoid allowing non-salient entities to become salient because of the conception of the constituency relations we have chosen. But the mereological conception does make them salient. In fact, the mentioned n-tuple of atoms is represented by the same object which represents the class of the two atomic facts, which are constituted by those atoms if the two states of affairs subsist: and this class of facts is just a molecular fact, according to the conception of molecular facts I chose in point 2) of my review of non-standard entities. Therefore, a molecular fact is represented by the same aggregate representing another uninteresting n-tuple of atoms: this is a further unwelcome consequence.

A third problem arises concerning the notion of world; it is the problem I have anticipated quoting the letter from Frege. *Tractatus* maintains explicitly and emphatically that:

1.1 The world is the totality of the facts.

Let us now identify the representation of the Tractarian world from a mereological point of view. The mereological representation of the world will include:

1. the atoms, as components of the atomic fact, which are members of the world (and the atoms are just the things excluded by section 1.1);
2. the above mentioned, non-salient non-atomic sub-parts of elementary facts;
3. any molecular fact.[12]

[12] This last consequence can be accepted and also welcome, if 'facts' in section 1.1 is meant to refer to all kinds of facts, both atomic and molecular. However, this reading of 1.1 is debatable: since the text refers to 'the totality of facts', it would be natural to think to a totality of mutually disjoint entities; on the contrary, molecular facts have other facts as their parts and do often overlap; therefore, in order to go from the totality of so many facts to the world, we should pass through something like the set theoretic union. Perhaps the problem can be settled only through a specification of the notion of mereological sum, which would be beyond the scope of the present paper.

Therefore, the world would acquire three problematic kinds of citizens, and one of these kinds is explicitly excluded by *Tractatus* itself.

It is possible to point to further odd consequences of the mereological approach. Let us make a supposition concerning the world: we can suppose that each atom is included in an atomic fact, i.e. that there is no atom such that its logical space is empty. In fact, it is true that *Tractatus* maintains that the logical space of an atom can be empty, but this does not imply that there is an object whose logical space is actually empty. Therefore, nothing in *Tractatus* excludes such a configuration of the world. What would the representing aggregate of such a world contain as components? Not only the members of the members of the world, i.e. all the atoms, but also anything which can be built combining together these atoms. But, according to *Tractatus*, the totality of atoms is co-determined with the totality of the states of affairs, including the non-subsistent states of affairs (*Tractatus* 5.5561): thus, conjoining atoms, we obtain all the states of affairs. But the totality of the states of affairs – subsistent and non-subsistent – is the logical space. Thus, world and logical space would be represented by the same aggregate, whereas these entities are clearly and emphatically distinguished all through *Tractatus*.

This list of problems is enough to conclude that mereology is an unsuitable tool to conceive and formalize the Tractarian ontology. The problems of formalization can perhaps be partially solved by replacing the transitive relation of 'being part of' with a suitable non-transitive relation, but the incompatibility between mereology and the Tractarian ontology is deeper. Frege's remark emphasized a crucial aspect of the Tractarian ontology.

A possible general objection can be envisaged. This objection does not concern the choice between set theory and mereology, but the relevance of my general approach. In fact, in some points of my comparison I assume that it is possible to formulate adequate identity conditions for complex entities by listing their constituents. This presupposition can be labeled as 'extensionalism' and could be disputed from several point of views.

In general, extensionalism is presupposed only when, applying the mereological machinery to constituency relations, we conclude that two ontological items come to have the same constituents and are therefore identified, while this identification contradicts an explicit Tractarian thesis or another well grounded expectation. In these cases, the extensionalist point of view is assumed in the implication from the fact that two complex entities have the same constituents to the fact that they are identical: if the

extensionalist presupposition is denied for the involved complex entities, then this implication is groundless.

I can not discuss here all the points of view on the Tractarian ontology connected with a certain skepticism towards an extensionalist approach to the Tractarian ontology. I mention only three broad Tractarian points, which could be raised against extensionalism, and try to specify the views on these issues which are actually incompatible with an extensionalist approach to the Tractarian ontology.

VI. Relations

Certain readings of the Tractarian theory of relations seem incompatible with the extensionalist presupposition. The Tractarian theory of relation is the object of the articulate exegetical debate, which can not be reviewed or summarized here. I can only stress that an extensionalist approach to the Tractarian ontology is compatible with more than one understanding of the Tractarian relations.

We can roughly distinguish five possible readings of the Tractarian theory of relations.

(1) relations are nothing (relation should be eliminated);
(2) relations are objects among others;
(3) relations are constituents of states of affairs, but not objects;
(4) relations are reducible ontological items which supervene on objects (e.g., through the compositional result of their form);
(5) relations are non-constituents which do not supervene on objects.

In cases (1), (2), (3) and (4), an extensionalist view on constituency and 'being in' is fully legitimate. In cases (2) and (3), relations are constituents among others: thus they should be mentioned when formulating the identity conditions for facts and states of affairs. In cases (1) and (4) we do not need to mention relations when defining identity conditions for states of affairs and atomic facts, since they are not irreducible constituents. Only the fifth view is definitely incompatible with an extensionalist approach.

VII. Logical forms

According to some scholars, the mere list of constituents (including eventually relations) is not enough to identify facts or states of affairs. The same class of constituents can be connected in different ways, generating different facts and states of affairs. These ways can not be reduced to a constituent: as Russell would have said (Russell 1903, §54), the unity of proposition is always lost when we provide a mere list of its constituents (and introducing new constituents is pointless).

These different forms of combination would be the logical form of the states of affairs (and the structure of a fact). If one subscribes to this reading of the Tractarian notion of the logical form of a state of affairs, one will not accept the extensionalist presuppositions of some arguments in this paper.

However, it should be stressed that there are many other readings of the Tractarian notion of logical form: some of them are compatible with extensionalism. Firstly, many scholars identify the logical form of a state of affairs with the relation in it: in this case, the compatibility with extensionalism follows from a choice concerning the nature of relations, as seen above. Secondly, some other scholars, while denying that the logical form of a state of affairs is identical with the relation, admits that it can be reduced to something else, e.g., to the logical forms of the constituents; thus, given a list of constituents, the logical form of the state of affairs including them is determinate. In this case,the logical form does not need to be mentioned in the identity conditions of the state of affairs and extensionalism can be endorsed.

VIII. Propositions as classes of names

Should one be worried by the following Tractarian sections, when systematizing the Tractarian ontology?

> *Tractatus* 3.14: The propositional sign is a fact.
> *Tractatus* 3.142: Only facts can express a sense, a class of names cannot.
> *Tractatus* 4.22: The elementary proposition consists of names. It is a connection, a concatenation of names.

If the names are the constituents of propositions and if 'fact' is here used in the specific sense of the Tractarian ontology (this second condition can be

questioned, but I will not question it here), then 3.14 and 3.142 imply that at least some facts are not classes of their constituents. This could be seen as an obstacle to an extensionalist approach to atomic facts in general.

My proposal is that a peculiar notion of class is being used here. In light of such a notion, facts are not classes, since the constituents of facts need to be connected by a relation. Relations can then be conceived in the five ways listed above. But, in any common notion of class, if the members of a class are connected by a relation, we can continue to call that class a „class". In other words, when we have a complex and its constituents, it may happen that the complex is not a mere class of its constituents, but it is anyway sound to say that it is a class of its constituents.

On the contrary, Wittgenstein seems to think that a class whose members are connected by a relation is not a class. In *Notebooks 1914-1916*, you can find some remarks about a comparison between a 'theory of the class' and a 'theory of the proposition' (Wittgenstein 1961a, 6.6.1915), where it is clear that the reason why a proposition is not a class of names is that, while a proposition is actually constituted by names, these names are connected by a relation. These difficult remarks can not be discussed here.

I quote only an interesting excerpt from Wittgenstein's review of Peter Coffey's *The Science of Logic*:

> He [Coffey] confounds classes and complexes. (Mankind is a class whose elements are men; but a library is not a class whose elements are books, because books become parts of a library only by standing in certain spatial relations to one another – while classes are independent of the relations between their members.) (Wittgenstein 1913).

The utterance 'a library is a class whose elements are books' can be meant in two ways: as an adequate definition or as a mere utterance. We should say that being a class of books is a necessary but not a sufficient condition of being a library, and so, that a library can not be adequately defined as a class of books (there are classes of books which are not libraries). However, in our notion of class, the utterance 'a library is a class of books' is plausibly true (it expresses a necessary condition of being a library).

Can the identity conditions for a library mention only the books constituting it? The answer to this question depends on the relevance of the role of relations: so, again, the viability of an extensionalist approach to constituency problems in *Tractatus* hinges on the role of relations. Therefore, this third theme can be reduced to the first one and does not constitute an independent argument against extensionalism.

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