# THE STRONG VERSION OF UNDERDETERMINATION OF THEORIES BY EMPIRICAL DATA: COMMENTS ON WOLEŃSKI'S ANALYSIS

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**Abstract:** The Polish researcher in the field of logic and philosophy, Jan Woleński, in one of his recent articles, „Metalogical Observations About the Underdetermination of Theories by Empirical Data," logically formalized two weak and strong versions of the underdetermination of theories by empirical data (or UT by abbreviation) and with these formalization has metalogically analyzed these two versions. Finally he has deducted that the weak version is defensible while the strong version is not. In this paper we will critically study Woleński's analysis of the strong version of UT.[1]

## I. Introduction

Although there have been many critics of the UT, but there is still enthusiasm towards it. It should be said that this enthusiasm is more likely to be a necessity, rather than a scientific curiosity. In fact, because some philosophers of science believe that the UT is a serious threat to realism (Devit 2005, pp. 761-791) some realist philosophers want to deny the possibility of the UT. Jan Wolenski in one of his recent papers attempts to refute the strong version of UT, by a metalogical approach (Woleński 2003, pp. 173-178).

In that paper, Woleński takes Mary Hesse's interpretation of UT and wants to rewrite and formalize it in the language of mathematical logic. In

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the next step, this formalization will be the base of his analysis of UT. He uses two concepts in mathematical logic: the branchability of a consistent set of sentences[2] like *X* at a sentence like *A*, and the Lindenbaum Lemma. With these two concepts he wants to show that, contrary to the weak version, the strong version of UT is absolutely not defensible and acceptable.

In this paper we will focus on a portion of Woleński's argument about the strong version of UT. Many philosophers of science believe that the weak version of UT can be shown to be consistent with realism, but there is no way to show the consistency of the strong version with realism (Devitt 2005). In Woleński likewise seeks to deny the strong version, as a threat to realism; but we think he has not succeeded in doing so (see. Woleński 2003). In the first section of this paper we will show Hesse's interpretation of the strong version of UT and Woleński's formalization of her interpretation. In the second section we will describe Woleński's argument. As we will show, this argument is unacceptable and has a big bug. Finally we will critically study Woleński's formalization. We believe that although this formalization has not been able to fulfill Woleński's purpose (i.e. to obviate the threat of the strong version of UT to realism), it can open new horizons for the understanding of UT.

## II . Hesse's Interpretation and Woleński's Formalization

Hesse interprets the strong version of UT thus: „For any given theory $T$, which is acceptable on the basis of some body of evidence $E$, there is at least one other incompatible and acceptable theory $T'$, which is empirically equivalent to $T$" (Hesse 1980, pp. 3-22). In her paper, Hesse has a detailed argument on the meaning of the empirical equivalency of two theories; but Woleński, begins by formalizing Hesse's views, without paying attention to this argument. In this formalization he views observations and theories as sets or collections[3] of propositions, and uses these concepts to logically[4] interpret the strong version of the UT.

[2] In this paper we do not consider any difference between the meaning of „sentences" and „proposition."

[3] The difference in the meaning of „set" and „collection" does not play an important role in this paper.

[4] In this paper by „logic," we mean „mathematical logic."

**Definition1.** A *theory* $T$ is a collection of sentences with the property $T \mapsto A \Rightarrow A \in T$ (see p. 106 from [5]).
According to this definition, a theory is *closed* under *derivability*.

**Definition2.** The smallest closed[5] superset of $\Gamma$ is named the *closure* of $\Gamma$ and is showed with $Cn\Gamma$.

**Definition3**. Theory $T$ is *finite axiomatizable* when for a finite *n*, there are $a_1, a_2, ..., a_n$ in $T$ that T = Cn $\{a_1, ..., a_n\}$. In this case $a_1, a_2, ..., a_n$ are the *axioms* of theory *T*.

Throughout his entire argument, Wolenski cites *scientific theories* as *finite axiomatizable theories* i.e. what has already been mentioned in definitions 1 and 3. Of course it must be said that he supposes that the axioms of scientific theories are certain sentences with a universal quantifier (i.e. $\forall$ ). Additionally he takes $KAX^{TH}$ as a symbol for the conjunction of the axioms of the finite axiomatizable theory *TH*; and he uses it to formalize Hesse's Interpretation in the language of mathematical logic[6]:

**TU1**: For any theory *TH* acceptable on the basis of [a collection of observational propositions] *E*, there is another theory $TH'$ such that:

(a) $E \subset CnKAX^{TH'}$;
(b) for every $E$, $E \subset CnKAX^{TH}$ if and only if $E \subset CnKAX^{TH'}$;
(c) *TH*, $TH'$ and $E$ are internally consistent;
(d) $TH \cup E$ and $TH' \cup E$ are [internally] consistent;
(e) *TH* and $TH'$ are mutually inconsistent (see P.174 of [7]).

Briefly it should be said that (b) shows the empirical equivalency of the theories *TH* and $TH'$, and (e) shows the inconsistency of these two theories. One of the first questions which may be asked about this formalization is the following, „Is the *theory-ladenness of observations* consistent with the interpretation of the set of observational propositions, as a common set *E*, in both theories *TH* and $TH'$?" (To know more about *theory-ladenness of observations*, see Hanson, Nowood Russell 1958). Wolenski claims that if we „admit that $A$ [which $A \in E$ ] has one meaning in the context of *TH*,

[5] We mean closed *under derivability*.
[6] Forthcoming we will use Wolenski's symbols.

but another one with respect to $TH'$, […] it raises the problem how $TH$ and $TH'$ can be empirically equivalent in this situation, because they work with different data" (Woleński 2003) Of course in the case of this formalization other questions are raised, e.g. „is (b) really a good interpretation of the empirical equivalency of scientific theories?" or „are all scientific theories finite axiomatizable?" Without exact consideration of such questions we cannot accept Woleński's formalization of the strong version of UT; but in the next section of this paper, without any challenges, we will implicitly accept Woleński's formalization and criticize the consequences derived from it.

## III. Wolenski's Deduction

In this chapter we will illustrate Woleński's deduction and for this, we have to define the concept of branchability:

**Definition4.** A set $X$ of sentences is *branchable* at sentence $A$ when the sets $X \cup \{A\}$ and $X \cup \{\neg A\}$, are both consistent.

$KAX^{TH}$ is the conjunction of sentences with universal quantifiers, and hence it can not be deduced from $E$ (because $E$ is a collection of observational propositions and hence without the universal quantifier); therefore we can conclude that $E \cup \{\neg KAX^{TH}\}$ is consistent. On the other hand $TH$ is consistent and $CnE \subset CnKAX^{TH} = TH$, thus $\neg KAX^{TH} \notin CnE$. This means that $E \cup \{KAX^{TH}\}$ is also consistent. Finally $E$ is branchable at $KAX^{TH}$ by definition. By similar reasoning it can be shown that $E$ is also branchable at $KAX^{TH'}$. Woleński claims that with these preliminaries we can deduce that for each proposition $A \in E$, $A \in Cn\ \{KAX^{TH}\}$ and $A \in Cn\ \{KAX^{TH'}\}$. We will show that his second result (i.e. $A \in \mathrm{Cn}\{\neg KAX^{TH'}\}$) is wrong; but to explain Woleński's purpose we suppose that they are both right. Obviously $A \in \mathrm{Cn}\ \{KAX^{TH'}\}$ because $A \in E \subset CnE \subset CnKAX^{TH'} = TH'$; this means that if $A \in \mathrm{Cn}\ \{\neg KAX^{TH'}\}$ then $\mapsto A$ and hence „$A$ is a theorem of logic and cannot represent any piece of synthetic empirical data" (in the words of Woleński) (Woleński 2003)

Woleński means that if TU1 is true then $E$ is necessarily a set of *tautologies* in the sense of Wittgenstein (Wittgenstein 1961) or a set of *analytic a priori* sentences in the sense of Kant (Kant 1999), whereas $E$ is a collection of observational propositions and its members should be synthetic empirical (or *a posteriori*) propositions. Therefore the strong version of UT, in the sense of its defenders, will not hold. Woleński argues about the probable answers to this dilemma and at the end he concludes that the strong version of UT is rejected. But as we mentioned above, Woleński's argument is not correct and we can find a counterexample for it. Suppose that in our experiments we have observed $A(c)$ (i.e. $E = \{A(c)\}$) and based on this observation we have presented two theories like $TH$ and $TH'$; such that:

$$AX^{TH} = \{\forall x A(x), \forall x B(x)\}$$

$$AX^{TH'} = \{\forall x A(x), \forall x \neg B(x)\}$$

It is clear that $E$, $TH$ and $TH'$ satisfy the conditions mentioned in TU1, and it is also clear that $A(c) \in Cn\ \{KAX^{TH}\}$ but there is no reason for the validity of $A(c) \in Cn\ \{\neg KAX^{TH'}\}$. Suppose the latter is valid then it must be that $A(c) \in Cn\{\exists x \neg A(x) \vee \exists x B(x)\}$; simply by choosing a proper predicate of $B$, we can appoint such conditions so that the latter relation will not be valid. This counterexample clearly shows that Woleński's argument of proving the analyticity – being a logical theorem – of members of $E$ is not proper.

## IV. Woleński's Formalization and Realism

Woleński implicitly assumed in his paper that propositions can be divided into observational propositions and non-observational propositions. An observational proposition asserts that an in principle observable event is occurring in a specified individual region of space and time. On the other hand the occurring of each, in principle observable, event produces an observational proposition which asserts this fact. This definition is clearly independent of the truth value of the propositions.

Now assume that we have two empirically equivalent theories based upon a set of observations (or more precisely: observational propositions) like $E$ and these two theories are valid under the TU1 conditions. Consider that $A$ is an observational proposition. It might be that the fact $A$ refers to (or its negation) is either observed or not; but because $A$ is observational that fact is in principle observable. TU1 asserts that $TH$ and $TH'$ are empirically equivalent and this property does not change over time; i.e. based on new observations, it would be impossible to prefer one of these theories to another. This means that by relying on (b) of TU1 the position of *TH* and *TH'* relating to *A* must be alike. In other words $A \in TH$ *iff* $A \in TH'$ (for each observational proposition $A$). Thus the inconsistency of *TH* and *TH'* will appear only in non-observational propositions. Now this raises the problem of what is the threat of such a non-observational proposition to realism?

Consider that the inconsistency of *TH* and *TH'* has appeared in $B$ which according to the above argument is a non-observational proposition. Based on (e) of TU1, suppose that $B$ is derivable from *TH* and its negation is derivable from *TH'*. It is probable that $B$ is one of the four types below:

1. $B$ is an analytic proposition: This judgment is contradictory to (c) of TU1 and therefore not acceptable. Because if $B$ is an analytic proposition then $\neg B$ is equivalent to $\perp$ and *TH'* should be internally inconsistent.

2. $B$ is a non-analytic physical proposition: In this case we would naturally ask what is the meaning of propositions which are non-analytic physical but *in principle* unobservable? Do we have such propositions?

3. $B$ is a non-analytic non-physical proposition: In this case we would naturally ask what is the threat of a non-physical proposition to scientific realism? It seems that if the inconsistency of *TH* and *TH'* appears only in non-analytic non-physical propositions then TU1 does not threaten scientific realism.

4. $A$ may be another type of proposition different than the above three types, of which we do not know.

It seems that by determining type $A$ and also giving an answer to the question proposed above, we can reach a new understanding of the strong version of UT and its relation with scientific realism.

## V. Epilogue

By relying on the arguments mentioned in this paper it should be said that even though there is great controversy concerning Woleński's formalization of the strong version of UT and although his argument for denying the strong version, is questionable, his formalization does open new horizons for an understanding of the strong version of UT. These horizons that may help us the solve the inconsistency of realism and the strong version of UT.

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