# THE RELATIONAL LOGIC OF FRANCISCUS TOLETUS AND PETRUS FONSECA

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**Abstract**. The well-known *Ratio Studiorum* of 1599 states that logical instruction should follow F. Toletus (Toledo) or P. Fonseca. The latter authored the famous *Institutionum Dialecticarum Libri Octo* (1564), the former a similar manual, *Introductio in Dialecticam Aristotelis* (1561). As is often observed, the contrast between the Aristotelian and present symbolic logics is perhaps most striking in their analysis of relational statements. Both authors recognize the relational logical form as independent from the traditional subject-predicate form and see the need to recognize relational inferential rules. They differ in their specific rules, however, so neither of the authors has captured the system of relational syllogism in its entirety.

As is well-known, the initial year of the three-year philosophical curriculum of Jesuit intellectual training was traditionally devoted to logic. In the third decade after its foundation, the early Society produced two outstanding textbooks of the discipline, those of Franciscus Toletus (Toledo) and Petrus Fonseca, which found their way to students in Bohemia during the 1570s[1] and which gained official recognition through the *Ratio Studiorum* of 1599. In the latter document one of the rules the professor of philosophy was required to follow reads as below:

(9.1.) He should explain the principles of logic the first year, devoting the first two months to a digest of it, not by dictating but by discussing pertinent passages from Toledo or Fonseca.

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[1] Sousedík draws our attention to notes which Edmund Campion took during his logic lectures of 1578–1579 in Prague, which are based on Toledo. See Sousedík, S., 1997, *Filosofie v českých zemích mezi středověkem a osvícenstvím* [Philosophy in the Czech Lands since the end of the Middle Ages until the period of Enlightenment], Prague: Vyšehrad, note 8, p. 69.

In this paper, I would like to introduce the two authors and their logical work, then compare their introductions to logic and, finally, analyze a particular aspect which is present in both of them, i.e. the logic of relational statements. The latter topic is especially interesting from today's perspective, for it is the logical analysis of relational statements that produced the greatest dissatisfaction with traditional logic as incapable of providing a logically satisfying treatment and the impetus to look for an alternative, ultimately giving rise to mathematical logic as we know it today.

Franciscus Toletus, SJ (Francisco Toledo; 1532–1596) was a Rome-based Spanish Jesuit and cardinal, a prolific writer in philosophy and theology both speculative and biblical (i.e. commentary on Aquinas and biblical commentaries). Apart from work on Aristotle's physical writings, he discussed his logic (*Commentaria una cum quaestionibus in universam Aristotelis logicam*; Rome, 1572) and wrote the aforementioned introduction to the discipline entitled *Introductio in Dialecticam Aristotelis*; Rome, 1561).

Petrus Fonseca SJ (Pedro da Fonseca or Pedro de Fonseca; 1528–1599) was known as the 'Portuguese Aristotle' for the depth and breadth of his learning and his influential commentary on Aristotle's *Metaphysics* (*Commentariorum in Libros Metaphysicorum Aristotelis Stagiritae*; Rome, 1577), perhaps overshadowed in importance only by the later *Disputationes metaphysicae* (1597) of Franciscus Suarez. Serving as a provincial of the Portuguese province he was the leading figure among the Conimbricenses, a group of scholars at the University of Coimbra who produced highly-prized commentaries on various works of Aristotle, combining scholastic speculative vigor with humanistic philological erudition. Among others, Fonseca is credited with inventing the teaching on *scientia media* or middle knowledge in God which was to play a key role in the well-known controversy over the nature of grace at the turn of the century associated with its use in a controversial work of another Jesuit, Louis de Molina, which came under attack by certain Dominican scholars. Fonseca was also an important figure behind the drafting of *Ratio Studiorum*. But it is to his extremely influential introduction to logic *Institutionum Dialecticarum Libri Octo* (Lisbon, 1564) that we now turn our attention.

The task is to compare both introductions to logic. We will be looking at Fonseca's *Institutionum Dialecticarum Libri Octo* (Cologne 1591 edition) and Toletus' *Introductio in Dialecticam Aristotelis* (Cologne 1574 edition) made available to the author. Let us set the contents of both works side by side and note similarities and differences (Fig.1).

Fig. 1

Comparison (as to content) of
Fonseca's  Institutionum Dialecticarum Libri Octo (Cologne 1591 edition)
Toletus' Introductio in Dialecticam Aristotelis (Cologne 1574 edition)

Toletus
- Liber I – basic explanation of logical terminology (logic, definition, argument, terms, statements…)
- Liber II – properties of terms and the doctrine of opposition and equivalence of statements
- Liber III – modal and other types of statements (exceptive, reduplicative, exponible, comparisons and superlatives)
- Liber IV – theory of syllogism (including modal)
- Liber V – topics and fallacies

Fonseca
- Liber I – theory of names (de nominibus)
- Liber II – categories, universals
- Liber III – theory of statements (de enuntiatione)
- Liber IV – theory of distinctions and divisions
- Liber V – theory of definition
- Liber VI – theory of consequence or argument (de consequentia)
- Liber VII – on demonstration and topics
- Liber VIII – sophisms and fallacies, theory of supposition

As regards similarities, both treatises are divided into books (*libri*) and follow the standard order of exposition based on Aristotle's *Organon*, i.e. the corpus of his logical writings: terms (*Categories*), statements (*On Interpretation*), syllogisms (*Prior Analytics*), theory of demonstrative argument and science (*Posterior Analytics*), fallacies (*On Sophistical Refutations*), theory of dialectical argument (*Topics*). This is only to be expected, as the authors are, after all, explaining Aristotelian traditional logic here. Both authors include material which goes beyond Aristotle and constitutes especially the late medieval addition to his logic, the doctrine of the properties of terms.

There are interesting differences to note: while Toletus gives some information on various properties of terms immediately after his general treatment of terms (book I) and before explaining various properties of statements in book II (i.e. material covered in Aristotle's *On Interpreta-*

*tion*) – this being a logical order of exposition – Fonseca specializes in the most important of the properties of terms, that of supposition, and attaches this treatment to his exposition of fallacies in the very last book (book VIII). This is understandable, as ignorance of the various suppositions often gives rise to fallacies. Further, it is interesting to note that Toletus assigns a separate book (book III) to special types of statements, especially modal, thus giving this material a certain prominence through conferring on it the status of a special book. His treatment of argument is standard, however, i.e. that of various types of syllogism. In contrast, Fonseca's book VI is conceived broadly as the theory of consequence, rather than the theory of the syllogism only. The latter is significant, for it seems that Fonseca generalizes both syllogism and other types of inference (those dealt with in contemporary propositional logic) under one heading rather than reducing all types of argument to syllogistic reasoning. This brings him more in line with today's approach. On the other hand, what seems a less progressive move from today's perspective is the inclusion of categories and universals in logic (book II) not found in Toletus – very much Aristotelian in character, because Fonseca enters the debate that has been associated with Aristotle's *Categories* ever since the influential ancient commentaries of Porphyry and Boethius – a topic properly treated in metaphysics today. Finally, the greater number of books (*libri*) in Fonseca's introduction in comparison with that of Toletus is the result of distinctions and divisions, the theory of definition and the treatment of demonstration being made into special books in Fonseca.

We are now in a position to examine the relational logic of both Jesuit writers. In order to make the most profitable comparison of this aspect of their respective works, we will outline their rules of relational syllogism against the background of the rules given by Juan Caramuel Lobkowitz (1606-1682), a Spanish Cistercian and the author of the greatest scholastic achievement in the field. His system of relational, i.e. oblique (or discrete, as he calls it) logic, written in Prague during Caramuel's years as abbot of the Benedictine Emmaus Monastery, appeared some seventy or eighty years later (in *Logica Obliqua*, part of the logical magnum opus *Theologia Rationalis*, Frankfurt, 1654).[2]

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[2] Dvořák, P., 2006, *Jan Caramuel z Lobkovic. Vybrané aspekty formální a aplikované logiky* [John Caramuel Lobkowitz: selected aspects of formal and applied logic], Prague: Oikumene. Dvořák, P., 2008, „Relational Logic of Juan Caramuel", in D. M. Gabbay, J. Woods (eds.), *Handbook of the History of Logic*: *Mediaeval and Renaissance Logic*, Volume 2, Amsterdam: North-Holland, pp. 645-666.

The so-called relational syllogism consists of statements which are themselves relational in character, i.e. include a complex predicate consisting of a transitive verb and a noun as its grammatical object. This noun, often quantified (if it is a general term), grammatically assumes a form other than the nominative in Latin, the so-called *casus obliquus* (oblique case) as opposed to the nominative, *casus rectus* (upright case). Hence, it is called an oblique term (*terminus obliquus*). Thus, relational statements – typically binary relations in today's understanding – are called oblique (*propositio obliqua*), for they contain an oblique term. For instance, in 'Peter is taller than John' the latter term (i.e. 'John') would be oblique. Consequently, relational syllogism is called oblique (*syllogismus obliquus*), for it contains such statements. For example,

Every A is taller than some B
Every C is taller than some A
Every C is taller than some B,

is oblique (here all constituent statements are oblique).

In contrast, standard Aristotelian categorical (or categorial) subject-predicate statements were 'upright' (*propositio recta*), containing only 'upright' terms (*termini recti*); e.g. 'Every man is mortal', the respective terms being 'man' and 'mortal'. These statements form the default 'upright' syllogisms (*syllogismus rectus*), i.e. two-premise arguments containing exactly three terms (the term appearing in both premises is called the middle, the other two, each one appearing in one of the premises and the conclusion, are called the extremes), dealt with in Aristotelian syllogistics. For example,

Every A is B
Every C is A
Every C is B.

Now the syllogistic rules to show validity were commonly provided or devised only for 'upright' syllogisms. The oblique syllogisms, as they stand, do not conform to these rules, for their form is deviant, containing a number of terms greater than three. The crucial question is the following: can these syllogisms be reduced to the 'upright' ones (via the reduction of their statements to 'upright' ones), without any logically relevant loss of their meaning, and consequently dealt with as if they were really 'upright' (then the standard rules could be applied to them), or not? The former option

was commonly advocated in scholastic logic. Yet, there are cases of oblique syllogisms which resist this type of reduction, at least without a substantial loss of meaning. So, it seems, one should prefer the latter negative option. If these syllogisms cannot be reduced, then some special rules have to be devised for them. Thus, the logic of relational or oblique syllogisms is recognized as different from the standard ones and special rules are provided. This progressive move comes late in scholastic logic and it is interesting to see that both our Jesuits give rules for showing the validity of oblique syllogisms in their introductions. Interestingly enough, it turns out that the rules set out are different and only when taken together approximate to a satisfactory set of rules.

In Toletus, one finds a short treatment of oblique syllogism in book IV dealing with the theory of syllogism (together with the so-called expository syllogism whose middle term is singular and is, consequently, in the third figure). In Fonseca, a short treatise on the topic constitutes a chapter with his theory of consequence or argument in book VI.[3]

Let us present the rules of both authors now. Fonseca gives four rules or precepts (*documenta*), Toletus explicitly gives three rules, yet two of them consist of two parts, so the overall number is greater. As it turns out, only two or three rule-schemes given by Fonseca are interesting, as they provide valid schemes of genuinely relational syllogisms. In comparison, Toletus presents four interesting schemes. Since the two groups of rule-schemes overlap only partially, it is evident that neither of the authors provides a full account of these interesting valid schemes. What makes a rule-scheme (logically) interesting? The answer is that it cannot be shown to be valid by the standard rules of 'upright' syllogisms. The rules of Fonseca will be presented in the form 'xF', those of Toletus 'xT' or 'xTy', where 'x' and 'y' are numbers. In contrast, Caramuel's rules will be numbered with no letters attached.

Fonseca's rule 1F: when the conclusion is affirmative: one relational premise leads to a relational conclusion.

If we reconstruct schemes from the examples provided by Fonseca we get the following:

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[3] Toletus, Liber IV, Caput 14 De Syllogismo ex obliquis et de Expositorio, pp. 158-159; Fonseca, Liber VI, Caput 30 De Syllogismis obliquis, pp. 297-302.

```
(i)  A a [RIb]      (ii)  A a [RIb]
     A ca                 A bc
     ----------           -----------
     A c [RIb]            A a [RIc]
```

where 'A' and 'I' are the quantifiers (every and some respectively), 'a', 'b' and 'c' are the terms used, 'R' is the term for relation (verb). The scheme (i) can be handled by standard rules for 'upright' syllogisms (syllogistic figure I, mood Barbara), because '[RIb]' appearing in the premise and the conclusion need not be analyzed into constituent parts at all and can be kept as one term – one of the extremes. This is so because the internal structure of [RIb] as such is not relevant for validity. In contrast, '[RI**b**]' and '[RI**c**]' are different and hence their internal structures need to be analyzed as relevant for validity. While (i) employs three terms ('a', 'c', '[RIb]'), (ii) employs four or more (either 'a', 'b', 'c' and 'R' or 'a', 'b', 'c', '[RIb]' and '[RIc]'). Hence, only scheme (ii) is relevant or interesting for relational or oblique syllogisms and constitutes an important specification of the rule 1F.

Caramuel provides four rules under Fonseca's rule 1F which I illustrate by means of schemes 1–4 below (*Logica Obliqua*, disp. VIII, pp. 430-432):

```
1. I/A a [RI/Ab]          2. I a [RAb]
   A bc                       I/A cb
   --------------------       ----------------
   I/A a [RIc]                I a [RI/Ac]



3. I/A a [nonRAb]         4. I/A a [nonRAb]
   I/A bc                    I/A cb
   -------------------       ------------------
   I/A a [nonRIc]            I/A a [nonRA/Ic]
```

Caramuel's first rule (scheme) – corresponding to Fonseca's (ii) – is richer, for it includes other quantifiers as well (those in bold are missing in Fonseca). The rule or scheme 2 is in a different figure (see the position of the middle term 'b'). Schemes 3 and 4 differ from 1 and 2, among others, in employing a negative relational term ('nonR').

Toletus' rule 1T: if the relational term (within relational predicate) is one of the extremes, it has to remain the same (i.e. part of the relational predicate) in the conclusion.

This can be brought out in the following scheme:

```
A a [RIb]
A/I ca
------------
A/I c [RIb]
```

As one can see, Toletus' 1T is essentially the same as Fonseca's not particularly useful 1F (i), which is discussed above. Toletus' second rule, however:

Rule 2T: (2T1) if the relational term is in the middle term position in one premise, then the extreme conjoined to the relational predicate will be inferred in the conclusion as non-relational, whereas the other extreme will appear as part of the relational predicate.
  (2T2) if in both premises the middle is a relational term, both extremes will be inferred in the conclusion as non-relational;

namely, its first part (2T1)

```
A/I a [RAb]
A cb
------------
A/I a [RAc]
```

For example,

```
Science concerns every being
(Every) body is being
Science concerns every body
```

is essentially the same as the second rule-scheme given by Caramuel above. The second part of Toletus' 2T, i.e. (2T2), will be discussed below.

Rule 2F: when the conclusion is negative: if the relational premise is affirmative, there is no conclusion. If the relational premise is negative, there is a negative relational conclusion.

```
 (i)  A a [RIb]              (ii)  E a [RIb]
      E bc                         A ca
      ----------                   ------------
      *E ac/ E a [RIc]             E c [RIb]
```

Again, scheme (ii) is not interesting from the point of view of relational syllogistics, for [RIb] can be kept as one term.

The second part (3T2) of Toletus' third rule 3T is essentially the same as 2F (i) of Fonseca above:

Rule 3T: (3T1) if the relational term appears in the negative premise, while the affirmative premise contains a non-relational term, the conclusion will be relational.
(3T2) if the affirmative premise is relational, while the negative premise is non-relational, no conclusion can be deduced.

While (3T2) can be illustrated with the following scheme (note some differences in the quantification in the first premise compared with (i) under 2F above):

```
 I a [RAb]
 E bc
 -------------
 *E a [RIc]
```

the scheme associated with (3T1) is that of Caramuel's fourth rule above:

```
 A/I a [nonRAb]
 A/I cb
 ---------------------
 A/I a [nonRAc]
```

Rule 3F: When the conclusion is affirmative: if both premises are relational, there is no conclusion.

```
 (i)  A a [RIb]      (ii)  A a [RIb]
      A a [RIc]            A c [RIa]
      ----------           ------------
      A bc                 A c [RIb]
```

It is clear that both schemes are relevant, for the relational predicate structures are different. The crucial question to ask now is whether this rule given by Fonseca is correct. Take scheme (i): it says that

```
Every A has R to some B
Every A has R to some C
does not validate the conclusion that
Every B is C
```

which seems obviously correct. Consider the following:

```
Every human performs some sense perceptions
Every human performs some vegetative functions
Every sense perception is a vegetative function
```

In traditional Aristotelian anthropology no sense perception is a vegetative function.

Now scheme (ii) is not obviously invalid. Indeed, extending Caramuel's rule or scheme 1 (which he clearly allows because of the valid moods of relational syllogism he provides) we get the following scheme of a purportedly valid relational syllogism whose premises are both relational:

```
 1.  I/A a [RI/Ab]
     A  b [RIc]
     -------------------
     I/A a [RIc]
```

Based on (ii), the following syllogism would be invalid according to Fonseca:

> Every A is taller than some B
> Every C is taller than some A
> Every C is taller than some B

Now let us change the order of premises (which is no doubt possible) and we get:

> Every C is taller than some A
> Every A is taller than some B
> Every C is taller than some B

Now let us replace C with A, A with B and B with C in the above syllogism, which again is clearly possible:

> Every A is taller than some B
> Every B is taller than some C
> Every A is taller than some C

The syllogism we get now is of the type Caramuel gives as valid and corresponds to the extended scheme 1 given above. So there are some syllogisms valid under (ii) and Fonseca seems to be wrong! But we should not jump to hasty conclusions. 'Taller than' is transitive, but would the syllogism be valid also if a non-transitive relation, such as 'loves' or an intransitive relation, such as 'is the father of', were to appear in it?

> Every A loves some B
> Every C loves some A
> Every C loves some B
>
> Every A is the father of some B
> Every C is the father of some A
> Every C is the father of some B

It does not seem so. Hence, on a charitable reading, one can understand why Fonseca posits no conclusion if the rule (rule 3F) is stated in such general terms. One can also see that the extended scheme we attributed to Caramuel (which, in fact, he does not state as such) depends for its validity on the formal properties of R.

Rule 4F: when the conclusion is negative: if both premises are relational, there is a non-relational conclusion:

```
E a [RIb]
A/I c [RIb]
--------------
O ca
```

The scheme based on examples given by Fonseca can be analyzed under the standard rules of 'upright' syllogism (syllogistic figure II, moods Cesaro [has existential import] or Festino), so it is not particularly useful. Essentially the same rule is provided by Toletus by his (2T2):

(2T2) If in both premises the middle is a relational term, both extremes will be inferred in the conclusion as non-relational.

The examples provided are of the following scheme:

```
E a [RIb]
I/A c [RIb]
------------
O/E ca
```

Again, the syllogism can be seen as 'upright', that of the second figure, Festino and Cesare.

At least one useful rule (7) is given by Caramuel, who states two rules of his own under the rule 4F:

```
6. A a [RAb]          7. I/A a [RIb]
   A/I c [nonRIb]        A/I  a [nonRAc]
Or  E/O c [RAb]        Or   E/O a [ RIc]
   --------------------    ------------------------
   E/O ca               O bc
```

As is evident, Caramuel's rule-scheme 6 can again be shown to be valid with the standard rules for 'upright syllogisms', for '[RAb]' can be regarded as one indivisible term (syllogistic figure II, moods Camestres, Baroco).

To sum up the interesting rules of Fonseca:

| 1F (ii): | 2F (i): |
|---|---|
| A a [RIb] | A a [RIb] |
| A bc | E bc |
| ----------- | ---------- |
| A a [RIc] | E a [RIc] |

Toletus includes the second one and leaves out the controversial 3F (ii).

Toletus includes two additional interesting rules (i.e. those of Caramuel, rules 2 and 4):

| Caramuel 2 | Caramuel 4 |
|---|---|
| A/I a [RAb] | A/I a [nonRAb] |
| A  cb | A/I cb |
| --------------- | ------------------- |
| A/I a [RAc] | A/I a [nonRAc] |

So it seems that the accounts of Fonseca and Toletus differ. Neither is complete in itself, i.e. neither includes all the interesting rules, as the comparison with the later set of rules given by J. Caramuel shows. Nevertheless, compared with other scholastic accounts of logic at the time and earlier, the introductions under discussion present an important step towards a genuine, non-reductive logic of relations and thus can be seen as progressive and advanced for their time. This again proves the high competence of Jesuits in various fields of research and – as I attempted to show – in logic in particular.