Grim, Omniscience, and Cantor's Theorem

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Abstract Although recent evidence is somewhat ambiguous, if not confusing, Patrick Grim still seems to believe that his Cantorian argument against omniscience is sound. According to this argument, it follows by Cantor's power set theorem that there can be no set of all truths. Hence, assuming that omniscience presupposes precisely such a set, there can be no omniscient being. Reconsidering this argument, however, guided in particular by Alvin Plantinga's critique thereof, I find it far from convincing. Not only does it have an enormously untoward side effect, but it is self-referentially incoherent as well.

Keywords Cantor's theorem; Grim, Patrick; omniscience; self-referential incoherence; set theoretical paradoxes

Alvin Plantinga, in my opinion, has decisively shown that Patrick Grim's Cantorian argument against omniscience fails.[1] Not everybody, however, seems to agree.[2] In particular, Grim himself, despite confessing to having

1. See Alvin Plantinga and Patrick Grim, "Truth, Omniscience, and Cantorian Arguments: An Exchange," *Philosophical Studies* 71 (1993), doi:10.1007/BF00989730. As for Grim's Cantorian argument, its underlying set theoretical mechanism is presented in Grim, "There Is No Set of All Truths," *Analysis* 44 (1984), doi:10.1093/analys/44.4.206. Elaborations and specific applications to omniscience appear in Grim, "Logic and Limits of Knowledge and Truth," *Noûs* 22 (1988); Grim, "On Omniscience and a 'Set of All Truths:' A Reply to Bringsjord," *Analysis* 50 (1990, doi:10.1093/analys/50.4.271; Grim, *The Incomplete Universe: Totality, Knowledge, and Truth* (Cambridge, MA: The MIT Press, 1991); Grim, "The Being That Knew Too Much," *International Journal for Philosophy of Religion* 47 (2000), 10.1023/A:1004091304240; and Grim, "Impossibility Arguments," in *The Cambridge Companion to Atheism*, edited by Michael Martin (Cambridge: Cambridge University Press, 2007).

2. According to Laureano Luna, for one, "the argument is still living." Laureano Luna,

Forum Philosophicum 17 (2012) no. 2, 211–223
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"second thoughts" about it,[3] still appears to believe that his argument is part of a logical case against omniscience which is "as close to a knock-down argument as one ever gets."[4] In what follows, I will look anew at Grim's argument alongside Plantinga's response, thus trying to bring out more clearly what exactly is at stake. In addition, I will comment on some later manoeuvres on Grim's part to reinforce his case. All in all, I will argue that believers in omniscience have little to fear in this Cantorian regard.

To begin with, however, we need to clarify what we mean by "omniscience." Grim's argument presupposes (commonsensically enough) that omniscience entails knowledge of *all* truths. Let us assume as much. Thus, somewhat more precisely, if by a "proposition" we mean whatever has the property of being true or false,[5] we might define "omniscient" as follows:

(O) A being B is omniscient if and only if, for every proposition P, if P is true then B knows P and if P is false then B does not believe P.[6]

In fact, here we have a fairly uncontroversial definition, one which not only is "employed by many theists,"[7] indeed, one which "in all likelihood has been the one most widely held among theists,"[8] but one which is unanimously affirmed by the "classical tradition in philosophy of religion."[9] So,

"Grim's Arguments against Omniscience and Indefinite Extensibility," *International Journal for Philosophy of Religion* 72 (2012): 91, doi:10.1007/s11153-011-9301-x.

3. Grim, "The Being That Knew Too Much," 141.

4. Grim, "Problems with Omniscience" (PDF on Grim's personal website, accessed July 9, 2013): 18, http://www.pgrim.org/articles/omniscience9.pdf.

5. Here I want to be as non-committal as possible as to what truth essentially is. I simply assume that a truth is a propositional (i.e. claim-making) entity that fits into the overall scheme of things: be it the objective features of the world (as correspondence theorists might insist), an appropriately overarching set of truth-bearers (as coherence theorists might insist), or the useful end of inquiry (as pragmatists might insist). A falsity, by contrast, I assume to be a propositional entity that does not fit into such a scheme of things. It makes little sense, however, to speak about true or false "truth-bearers." Hence I adopt the term of art "proposition." Those claim-making things which are true or false, then, depending on whether or not they fit into the overall scheme of things, are propositions.

6. It is a matter of debate whether the clause "and if P is false then B does not believe P" is redundant or not. For contrasting views, see Grim, "A Reply to Bringsjord," 273; and Edward Wierenga, *The Nature of God: An Inquiry into Divine Attributes* (Ithaca; London: Cornell University Press, 1989), 38–39.

7. Joshua Hoffman and Gary S. Rosenkrantz, *The Divine Attributes* (Oxford: Blackwell, 2002), 112.

8. Tully Borland, "Omniscience and Divine Foreknowledge," *Internet Encyclopedia of Philosophy*, last modified July 7, 2006, http://www.iep.utm.edu/omnisci.

9. Gerard J. Hughes, *The Nature of God* (London and New York: Routledge, 1995), 64.

if perchance Grim's Cantorian argument succeeds, it follows by (O) that the God of classical western theism, who is believed to be essentially omniscient, cannot exist. There is some reason, then, given that there still seems to be some uncertainty as to its philosophical status, to look into this matter afresh.

Restating Grim's Argument

Grim's Cantorian argument can be presented in two stages. First, as an assumption for *reductio*, suppose that there is a set **T** of all truths (or true propositions): $\{T_1, T_2, T_3, \ldots\}$. Now according to Cantor's "widely accepted" power set theorem, every set has more subsets than elements.[10] In finite cases this is fairly obvious. Take, for instance, the set of pencils presently on my desk. It contains three items: $\{A, B, C\}$. Each of these elements forms a singleton (a set with exactly one member): $\{A\}$, $\{B\}$, and $\{C\}$. Then of course there are three subsets of pairs: $\{A, B\}$, $\{A, C\}$, and $\{B, C\}$. In addition, albeit less obviously, there are the empty set, $\varnothing$, and the entire set, $\{A, B, C\}$. So, whereas the set of pencils presently on my desk only has three members, it has no less than eight subsets. These eight subsets constitute the power set of the set in question. Generally speaking, using a set theoretical locution, a power set, $\mathscr{P}(S)$, is the set of subsets of a given set S. That is to say, the elements of $\mathscr{P}(S)$ are the subsets of S, and thus $\mathscr{P}$(the set of pencils presently on my desk) has eight elements. What Cantor's theorem shows is that given some suitable (and preferably not ad hoc) paradox blocking restriction, *all* sets—empty, finite, or infinite—have more subsets than elements. For any set, then, $\mathscr{P}(S)$ is larger in size (cardinality) than S itself.

But then it follows by Cantor's theorem that $\mathscr{P}(\mathbf{T})$ is larger, that is, contains more elements, than **T** itself. Paradoxically, however, as Grim points out, "to each element of this power set will correspond a truth."[11] For example, it will be true about every element of $\mathscr{P}(\mathbf{T})$ whether a certain truth, say, T_{18}, belongs to it or not. So, "there will be at least as many truths as there are elements of the power set $\mathscr{P}(\mathbf{T})$."[12] But **T** itself is supposed to contain *all* truths. Accordingly, or so Grim argues, since there obviously cannot be more truths than all truths, it follows by Cantor's theorem that, contrary to our initial assumption, there is in fact no such thing as **T**.

10. Kevin C. Klement, "Russell, His Paradoxes, and Cantor's Theorem: Part I," *Philosophy Compass* 5 (2010): 16, doi:10.1111/j.1747-9991.2009.00270.x.

11. Grim, "There Is No Set of All Truths," 207.

12. Ibid.

Having thus apparently established what "some" or even "many" think is "*strongly* counterintuitive,"[13] namely, that there cannot be any set of all truths, Grim proceeds to the second stage of his argument: a "short and sweet Cantorian argument against omniscience."[14] It goes thus:

> Were there an omniscient being, what that being would know would constitute a set of all truths. But there can be no set of all truths, and so can be no omniscient being.[15]

Short and sweet indeed, this argument "illustrates very neatly how debates about theism can be enriched by philosophical ideas coming from wholly unexpected directions."[16] If it is sound, omniscience à la (O) is impossible. Here, then, is where our inquiry begins. As we proceed, there are three points I would like to make.

Paradoxical Implications

Here is the first point—a familiar one at that, although it did not receive much attention in the published exchange between Plantinga and Grim. The paradoxical potential of Cantor's theorem is by no means a particular problem for theism and notions of omniscience. In fact, unless it is somehow held in check, "Cantor's law falls immediately into paradox."[17] To see this as clearly as possible, consider the set of all sets. Unless somehow restricted, Cantor's theorem entails that $\mathscr{P}$(set of all sets) contains more sets than the set of all sets itself, which is absurd. Or consider an even grander set: the absolutely universal set, U, of all things. Unless restricted, it follows by Cantor's theorem that $\mathscr{P}(U)$ contains more elements than U itself. Thus it is clear that Cantor's theorem must be somehow restricted in scope, or "set" has to be defined so as to exclude collections that are "too

13. J. C. Beall, "A Neglected Response to the Grim Result," *Analysis* 60 (2000): 38, doi:10.1093/analys/60.1.38.

14. Grim, "Logic and Limits of Knowledge and Truth," 356.

15. Ibid.

16. Nicholas Everitt, *The Non-Existence of God* (London and New York: Routledge, 2004), 288. For an earlier argument against the coherence of omniscience that is vaguely reminiscent of Grim's, see Roland Puccetti, "Is Omniscience Possible?," *Australasian Journal of Philosophy* 41 (1963), doi:10.1080/00048406312341561. Fred Newman's, "Omniscience Is Possible," *Australasian Journal of Philosophy* 42 (1964), doi:10.1080/00048406412341091, is a good reply.

17. Willard Van Orman Quine, *Set Theory and Its Logic* (Cambridge, MA: Belknap Press, 1963), 202.

big,"[18] or whatever—*some* paradox blocking approach must be pursued. In short: "naive" set theory must somehow turn "axiomatic."[19] Yet it is one of the great logical debates in modern times, ever since Cantor laid down his theorem in 1891, exactly how this ought to be done. As W. V. O. Quine opines, further research "may some day issue in a set theory that is clearly best," but, as they stand, "the axiomatic systems of set theory in the literature are largely incompatible with one another and no one of them clearly deserves to be singled out as standard."[20]

Let us dwell on this point for a while. What we have is a century-plus long deliberation at the highest logical level, dividing the world's leading set theoreticians and resulting in several alternative axiomatizations that are "largely incompatible with one another." Each version, as Grim says, "is essentially a response to two paradoxes: Cantor's paradox regarding a set of all sets and Russell's paradox regarding a set of all non-self-membered sets."[21] One of these versions, it should be noted, namely, the ZF (Zermelo-Frænkel) axiomatization, is "dominant" among mathematicians working today.[22] Notably, this commonly used version solves Cantor's set-of-all-sets paradox by "proving" (as it were)—from its careful selection of axioms—that this set cannot exist. In short, it turns the paradox into a *reductio*: if there *were* such a thing as the set of all sets then by Cantor's theorem this set would contain fewer sets than its power set. Hence, since this is absurd, there *is* no such thing as the set of all sets. And of course this is a parallel conclusion to Grim's less general one, with which we are already acquainted, namely, the conclusion that there is no such thing as the set of all *truths*. Not surprisingly, then, in ZF's well-documented mathematical utility Grim finds additional support.

But of course ZF set theory does not *prove* that the set of all sets, or the universal set of everything, does not exist. As M. Randall Holmes cautions,

> there is a good reason for mathematicians who have occasion to think about

18. Keith Simmons, "Semantical and Logical Paradox," in *A Companion to Philosophical Logic*, edited by Dale Jacquette (Malden, MA: Blackwell, 2002), 123.

19. Patrick Suppes, *Axiomatic Set Theory* (New York: Dover, 1972), 3.

20. Quine, *Set Theory and Its Logic*, viii.

21. Grim, "Logic and Limits of Knowledge and Truth," 358. Russell's famous paradox is generated by the following question: Is the set of non-self-membered sets a member of itself? If yes, it is not a member of itself. If no, it is a member of itself. In either case it is a member of itself only if it is not.

22. M. Randall Holmes, "Alternative Axiomatic Set Theories," *The Stanford Encyclopedia of Philosophy* (Winter 2012 Edition), last modified April 3, 2012, http://plato.stanford.edu/archives/win2012/entries/settheory-alternative.

> foundations to be aware that there are alternatives; otherwise there is a danger that accidental features of the dominant system of set theory [i.e. ZF] will be mistaken for essential features of any foundation of mathematics. For example, it is frequently said that the universal set . . . is an inconsistent totality; the actual situation is merely that one cannot have a universal set while assuming Zermelo's axiom of separation.[23]

Indeed, says Holmes, "[a] common criticism of Zermelo set theory is that it is an *ad hoc* selection of axioms chosen to avoid paradox."[24] Bertrand Russell, for one, "had many reasons for not finding this [size limitation] approach [of which ZF is an example] very attractive." For example, he "insisted upon an independently *philosophically well-motivated explanation*" of which sets one might assume to exist, rather than just dismissing those candidates that are "too big" to be consistent with certain theorems.[25] Here, however, I have no intention of trying to substantiate this common line of criticism. What is important to note is merely that, even if, pace Quine, ZF should be considered "clearly better" than its axiomatic rivals, it does not follow that the set of all sets, or the set of everything, is an impossible totality. To validate this metaphysical conclusion, some further argument, something "beyond anything Grim supplies,"[26] is needed.

Thus, to reiterate, the paradoxical potential of Cantor's theorem is by no means a particular problem for theism and notions of omniscience. As indeed Plantinga says, it is "a general problem with a life of its own."[27] Of course, as Grim correctly replies, this generality does not mean that it is *not* a problem for theism and omniscience.[28] But the correct lesson to be drawn so far is rather something like this: even if Cantor's theorem poses a problem for the idea of omniscience, as defined by (O), it does not warrant Grim's concluding speculation that, "within any logic we have . . . omniscience appears to be simply impossible."[29] As Selmer Bringsjord says, "[a] number of axiomatic set theories lacking the power set axiom are, according to many, genuine foundational contenders."[30] Also, as Holmes

23. Ibid.

24. Ibid.

25. Kevin C. Klement, "Russell, His Paradoxes, and Cantor's Theorem: Part II," *Philosophy Compass* 5 (2010): 30, doi:10.1111/j.1747-9991.2009.00271.x.

26. Keith Simmons, "On an Argument Against Omniscience," *Noûs* 27 (1993): 27.

27. Plantinga and Grim, "An Exchange," 287.

28. Ibid., 297.

29. Grim, "Logic and Limits of Knowledge and Truth," 341, see also 359.

30. Selmer Bringsjord, "Grim on Logic and Omniscience," *Analysis* 49 (1989): 187, doi:10.1093/analys/49.4.186.

notes, "many of the alternative set theories" aim to recover the universal set: a recovery that, if successful, authenticates the set of all truths along with it. And again, how best to axiomatize set theory has been discussed for more than a century. "[T]he last time I looked at these disputes," says Bringsjord, "they weren't settled—not in the least."[31]

Sets and Quantifications

The second point is largely due to Plantinga and goes as follows. Why think that knowledge of all truths presupposes the existence of a *set* of all truths? In their published exchange, this is the first thing Plantinga asks Grim:

> why do you think the notion of omniscience . . . demands that there be a *set* of all truths? As you point out, it's plausible to think that there is no such set. . . . So I'm inclined to agree. . . . But how does that show that there is a problem for the notion of a being that knows all truths?[32]

What Plantinga somewhat sparingly seems to suggest here is that although omniscience by (O)-definition involves knowledge of all truths, these truths need not form a ZF-defined, recursively enumerable entity called "set." It is perfectly possible to quantify over all truths (and indeed over *all* propositions) anyway, without therefore being committed to the additional existence of a mathematical ZF set. For example, the universal assertion that "every proposition is either true or not-true" is, well, true, despite the fact that Cantor's theorem seems to imply that there can be no set of all propositions. In short, then, as Plantinga nicely puts it, "why buy the dogma that quantification essentially involves sets?"[33]

In reply, Grim concurs that the "appeal directly to propositional quantification" is "clearly the most plausible response" to the said argument.[34] Nevertheless, there is (Grim continues) an "immediate problem" even with

31. Ibid.

32. Plantinga and Grim, "An Exchange," 267–68.

33. Ibid., 268. Discussing this "dogma," Richard L. Cartwright argues that "[t]here would appear to be every reason to think it false. Consider what it implies: that we cannot speak of the cookies in the jar unless they constitute a set. . . . I do not mean to imply that there is no set the members of which are the cookies in the jar. . . . The point is rather that the needs of quantification are already served by there being simply the cookies in the jar . . . no additional objects are required." Cartwright, "Speaking of Everything," *Noûs* 28 (1994): 8.

34. Plantinga and Grim, "An Exchange," 268.

this appeal, for "the only semantics we have for quantification is in terms of *sets*" and hence "even appeal to propositional quantification fails to give us an acceptable notion of omniscience."[35] What exactly Grim means by this is, unfortunately, not clear, but he seems to be suggesting something like this: take away the set theoretical framework and we are no longer able to understand what omniscience is supposed to involve.

Plantinga's rejoinder, however, is clarifying:

> If we think we have to employ the notion of set in order to explain or understand quantification, then some of the problems you mention do indeed arise; but why think that? The semantics ordinarily given for quantification already presupposes the notions of quantification; we speak of the domain D for the quantifier and then say that "(z)Az" is true just in case every member of D has (or is assigned to) A. So the semantics obviously doesn't tell us what quantification is.[36]

Indeed, if I might add something to this thought, the domain D, according to ZF set theory, is defined or "determined" by the entities which belong to it "in the sense that sets with exactly the same elements are identical."[37] So, in order to define D, one must first have identified all of its members. But if thus all its members are implicit in the definiens or determination of D then ZF set theory cannot be required to meaningfully grasp the notion of quantification. It is rather the other way around: "in a sense," as George Boolos says, "the elements of a set are 'prior to' it."[38]

Anyway, as Plantinga points out (again somewhat sparingly), had it been sound, Grim's argument would have had a remarkably adverse side effect. To see what Plantinga is after, we need to recall that Grim tries to use Cantor's theorem in order to validate the conclusion that there can be no set of all truths. By exactly parallel arguments, however, neither can there be a universal set, a set of all sets, or a set of all propositions—and indeed, according to ZF set theory, none of these sets exist. Hence if, as Grim argues, quantification semantically presupposes the existence of sets then none of the following sentences expresses any proposition (truth bearer) at all:

35. Ibid., 269. See also Patrick Grim, "On Sets and Worlds: A Reply to Menzel," *Analysis* 46 (1986): 191, doi:10.1093/analys/46.4.186.

36. Plantinga and Grim, "An Exchange," 271–72.

37. George Boolos, "The Iterative Conception of Set," *The Journal of Philosophy* 68 (1971): 215.

38. Ibid., 216.

(1) Nothing exists.
(2) All existing things exist.
(3) All false propositions are true.
(4) If P, and if P implies Q, then Q.
(5) There is no object with six legs.

But this is fantastically unbelievable. As just about anyone would agree, (1) is false, whether contingently or necessarily; (2), a tautology, is necessarily true; (3), a contradiction, is necessarily false; (4), modus ponens, is a non-negotiable axiom of classical logic; and (5) is false—albeit not ecessarily so.

Amazingly, conceding that Plantinga's argument "does raise a very important question as to what formal semantics can honestly claim or be expected to do,"[39] Grim seems to bite the bullet: "It must," he says, "be admitted that another casualty [of my Cantorian argument] would be 'logical laws' of the form you indicate," for example, the law of non-contradiction.[40] Apparently, then, Grim is prepared to accept the conclusion that sentences such as (1)–(5) lack truth values. To be willing to pay such a high price in order to save one's argument is, in a way, admirable, although it would be even better, of course, to draw the conclusion that there is something wrong with one's argument to begin with.

Self-referential Incoherence

As if this were not enough, there is another fatal flaw with Grim's Cantorian argument. This is the third point I want to make—it, too, is largely due to Plantinga.[41] Now it will be recalled that, according to Grim's argument, it follows by Cantor's theorem that there is no set of all truths. But note that this is a conclusion about *all sets*, namely, that no set is a set of all truths. That is to say, it is a kind of universal conclusion which on Grim's own account cannot have a truth value. Absurdly, had Grim's argument been sound, it thus follows that its conclusion would not have been true (since it would have lacked a truth value). Therefore, by *reductio*, Grim's argument is, quite palpably, unsound.

Interestingly, there have been some developments on this point. On Grim's later admission, it highlights "the one aspect of my earlier work

39. Plantinga and Grim, "An Exchange," 273–74.
40. Ibid., 271.
41. Ibid., 284–87 and 291–97.

about which I've come to have the gravest second thoughts."[42] In fact, judging by his most recent publication on this matter, "Plenum Theory," co-written with Nicholas Rescher, it would now seem to be all but official that he has retracted from his earlier claims. Speaking about such collections as the totality of all truths and the totality of all things, he explicitly points out that Cantor's theorem "affords no sufficient ground for deeming such mega-collectivities impossible, let alone logically inconsistent."[43] It is therefore all the more surprising that he still seems to maintain that "there can be no plurality of all truths," indeed, that the assumption of such a thing is "provably false,"[44] given "elementary logic."[45] Might it be possible, in spite of all that has been said so far, and in spite of what Grim himself and Rescher say in "Plenum Theory," to somehow reinforce his Cantorian case?

Well, already in his correspondence with Plantinga, Grim seems to think so. Admitting that "Cantorian arguments are indeed very peculiar, tempting us in some cases to try to draw universal conclusions that they themselves show us cannot be drawn,"[46] he suggests a different strategy, "less direct and more deviously dialectical."[47] According to this alternative strategy, it is possible to rephrase his Cantorian argument "*purely* in the particular, without any universal propositions at all."[48] More precisely, as Grim later develops it, the idea is that although the conclusions of his Cantorian argument (viz. that there is no set of all truths and hence no one who is omnipotent) "cannot be represented in the manner we might first attempt," the argument as such can still be directed "case by case" as a "logic bomb" against any particular affirmations of omniscience.[49] Thus, for example, should someone affirm the existence of an omniscient being, the Cantorian argument can be used to expose the incoherence of that particular claim. As Grim tries to argue already in his correspondence with Plantinga:

42. Grim, "The Being That Knew Too Much," 141.

43. Nicholas Rescher and Patrick Grim, "Plenum Theory," *Noûs* 42 (2008): 423, doi:10.1111/j.1468-0068.2008.00690.x.

43. Grim, "Impossibility Arguments," 208. This paper, it should be noted, is published in 2007, only one year before "Plenum Theory."

44. Grim, "Problems with Omniscience," 6. This hitherto unpublished paper, it should be noted, contains references to the published version of "Plenum Theory."

45. Ibid., 9.

46. Plantinga and Grim, "An Exchange," 299.

47. Ibid., 298.

48. Ibid., 299.

49. Grim, "The Being That Knew Too Much," 152.

50. Plantinga and Grim, "An Exchange," 298.

> Contrary to the characterization you [Plantinga] give, I'm not trying to get you to envisage and accept an argument with some universal premise and a universal conclusion to the effect that there are no universal propositions. You characterize yourself as holding certain beliefs. I merely help you to see that you are thereby led to confusion and consternation.[50]

In effect, however, this alternative, indirect, and "deviously dialectical" strategy is just as flawed as the original one. Let P be the particular claim that God is omniscient. So, according to Grim, the Cantorian argument can be somehow used to demonstrate the incoherence of P. But how is this demonstration supposed to work? By making the proponent of P see that P implies quantification over all truths, which implies that there is a set of all truths, which by Cantor's theorem implies that there are more truths than all truths, which is absurd—while at the same time *not* denying that there *is* a totality of all truths (since this would be to confusingly quantify over all objects) or a set of all truths (since this would likewise be to confusingly quantify over all sets)? But then of course the proponent of P can simply retort that, if indeed the objector does not deny either that there is a totality or a set of all truths, what is wrong with affirming P? After all, it is for the accuser to substantiate his claim.

More importantly, however, Grim's alternative strategy works equally well (or rather badly) against other particular claims, such as (1)–(5) above. Thus, for example, should we want to affirm that all existing things exist, it follows by Grim's alternative strategy that we shall thereby be led to "confusion and consternation." No less than on its original slant, Grim's Cantorian argument thus still seems to have an unacceptable side effect.

Conclusion

If there is a lesson to be learned from all this, it might be that theists and atheists are in the same boat. Those who affirm the existence of an omniscient being and those who deny it are all somehow relying on the possibility of universal (or at least enormously wide-ranging) quantifications. According to the former, at least those of whom accept definition (O), God's knowledge encompasses all truths. According to the latter, there is (among all objects) no one who is omniscient. In sum, then, if there is a Cantorian problem associated with the concept of omniscience, it presents itself to theists and atheists alike. Not an unexpected result, perhaps, given that the entire issue is fuelled by the paradoxical implications of Cantor's theorem.

Bibliography

Beall, J. C. "A Neglected Response to the Grim Result." *Analysis* 60 (2000): 38–41. doi:10.1093/analys/60.1.38.

Boolos, George. "The Iterative Conception of Set." *The Journal of Philosophy* 68 (1971): 215–31.

Borland, Tully. "Omniscience and Divine Foreknowledge." *Internet Encyclopedia of Philosophy.* Entry last modified July 7, 2006. http://www.iep.utm.edu/omnisci.

Bringsjord, Selmer. "Grim on Logic and Omniscience." *Analysis* 49 (1989): 186–89. doi : 10 . 1093/analys/49.4.186.

Cartwright, Richard L. "Speaking of Everything." *Noûs* 28 (1994): 1–20.

Everitt, Nicholas. *The Non-Existence of God.* London and New York: Routledge, 2004.

Grim, Patrick. "There Is No Set of All Truths." *Analysis* 44 (1984): 206–08. doi:10.1093/analys/44.4.206.

———. "On Sets and Worlds: A Reply to Menzel." *Analysis* 46 (1986): 186–91. doi : 10 . 1093 / analys/46.4.186.

———. "Logic and Limits of Knowledge and Truth." *Noûs* 22 (1988): 341–67.

———. "On Omniscience and a 'Set of All Truths:' A Reply to Bringsjord." *Analysis* 50 (1990): 271–76. doi:10.1093/analys/50.4.271.

———. *The Incomplete Universe: Totality, Knowledge, and Truth.* Cambridge, MA: The MIT Press, 1991.

———. "The Being That Knew Too Much." *International Journal for Philosophy of Religion* 47 (2000): 141–54. doi:10.1023/A:1004091304240.

———. "Impossibility Arguments." In *The Cambridge Companion to Atheism*, edited by Michael Martin, 199–214. Cambridge: Cambridge University Press, 2007.

———. "Problems with Omniscience." PDF on Grim's personal website. Accessed July 9, 2013. http://www.pgrim.org/articles/omniscience9.pdf.

Hoffman, Joshua, and Gary S. Rosenkrantz. *The Divine Attributes.* Oxford: Blackwell, 2002.

Holmes, M. Randall. "Alternative Axiomatic Set Theories." *The Stanford Encyclopedia of Philosophy* (Winter 2012 Edition). Entry last modified April 3, 2012. http://plato.stanford.edu/archives/win2012/entries/settheory-alternative.

Hughes, Gerard J. *The Nature of God.* London and New York: Routledge, 1995.

Klement, Kevin C. "Russell, His Paradoxes, and Cantor's Theorem: Part I." *Philosophy Compass* 5 (2010):16–28. doi:10.1111/j.1747-9991.2009.00270.x.

———. "Russell, His Paradoxes, and Cantor's Theorem: Part II." *Philosophy Compass* 5 (2010): 29–41. doi:10.1111/j.1747-9991.2009.00271.x.

Luna, Laureano. "Grim's Arguments against Omniscience and Indefinite Extensibility." *International Journal for Philosophy of Religion* 72 (2012): 89–101. doi:10.1007/s11153-011-9301-x.

Newman, Fred. "Omniscience Is Possible." *Australasian Journal of Philosophy* 42 (1964): 102–03. doi:10.1080/00048406412341091.

Plantinga, Alvin, and Patrick Grim. "Truth, Omniscience, and Cantorian Arguments: An Exchange." *Philosophical Studies* 71 (1993): 267–306. doi:10.1007/BF00989730.

Puccetti, Roland. "Is Omniscience Possible?" *Australasian Journal of Philosophy* 41 (1963): 92–93. doi:10.1080/00048406312341561.

Quine, Willard Van Orman. *Set Theory and Its Logic.* Cambridge, MA: Belknap Press, 1963.

Rescher, Nicholas, and Patrick Grim. "Plenum Theory." *Noûs* 42 (2008): 422–39. doi:10.1111/j.1468-0068.2008.00690.x.

Simmons, Keith. "On an Argument Against Omniscience." *Noûs* 27 (1993): 22–33.

———. "Semantical and Logical Paradox." In *A Companion to Philosophical Logic*, edited by Dale Jacquette, 115–30. Malden, MA: Blackwell, 2002.

Suppes, Patrick. *Axiomatic Set Theory.* New York: Dover, 1972.
Wierenga, Edward. *The Nature of God: An Inquiry into Divine Attributes.* Ithaca; London: Cornell University Press, 1989.